Think about the distribution of velocities in a gas in equilibrium in the following way. Let $F(v) \in C^1[R^3 \to R]$ be the probability density for the equilibrium velocity, so that $\int_D F d^3v$ is the probability that a typical velocity lies in $D \subset R^3$. Naturally, $F > 0$, $\int_{R^3} F = 1$, and it is natural to suppose $\int_{R^3} vF = 0$, i.e. the mean velocity is zero. Now the 3-dimensional volume element is rotation invariant and the velocity distribution should be so, too, i.e. $F = F(r), r = \sqrt{v_1^2 + v_2^2 + v_3^2}$. Also, in any frame, the 3 components of velocity should be statistically independent and since independence requires that probabilities multiply, F must split as in $F(r) = f(v_1)f(v_2)f(v_3)$. One final point: if m is the mass of a molecule, then $\int \frac{1}{2}m|v|^2 F d^3v$ is mean kinetic energy and ought to be proportional to temperature T, heat being the bulk aspect of molecular aggitation. It is conventional to write $\int |v|^2 F = 3kT$, in which k is Boltzmann's constant. Prove that F can only be the Maxwellian distribution

$$\frac{e^{-|v|^2/2kT}}{(2\pi kT)^{3/2}}$$

As this is an analysis question, please be sure to be rigorous and as detailed as possible. I would also prefer the solution in PDF format. Thank You.