In class we wrote down the thermodynamical potential for a gas of non-interacting (non-relativistic) fermions in three dimensions. We found a simple result for fermions of the form

$$\Omega = -k_B T \int_0^\infty g(\varepsilon) \ln[1 + e^{-\beta(\varepsilon-\mu)}] d\varepsilon \tag{1}$$

Here $g(\varepsilon) = \varepsilon^{1/2} 2^{1/2} V m^{3/2} \pi^{-2}$ is the usual density of states and V is the volume. Note there is an analogous expression for the Bose case (with two sign changes).

$$\Omega = +k_B T \int_0^\infty g(\varepsilon) \ln[1 - e^{-\beta(\varepsilon-\mu)}] d\varepsilon \tag{2}$$

We want to use Eqs (1) and (2) to derive the pressure P associated with a Fermi gas (and its analogue for the Bose gas). We will show that it satisfies $PV = (2/3)E$.

(1a) Prove that the relation $PV = (2/3)E$ is satisfied at all temperatures T for a *classical* ideal gas.

(1b) Derive the Helmholtz free energy F from the partition function (which is simply related to the grand partition function: $Z_g = Ze^{\beta\mu N}$, where $\Omega = -k_B T \ln Z_g$) to arrive at an expression for the pressure, via the usual derivative of F. Note that you must include $\partial\mu/\partial V$ in your analysis. Then prove that

$$P = -(\partial\Omega/\partial V)_{T,\mu}$$

(2a) Next, show that

$$P = -\Omega/V$$

(2b) Note that by integrating Ω by parts the equation of state is therefore

$$PV = (2/3)E$$

where E is the average energy of the system at arbitrary temperature. Note this is true for both Bose and Fermi gases. Hence, knowing the ground state energy is sufficient to obtain the ground state pressure.