Please show detail

ODE:

1. Solve $y' = \sin(x+y)$.
2. Find the complete solution of the ODE $y^{(4)} - y^{(2)} - 2y = 12x\cos x$.
3. Find the complete solution of the ODE $y^{(4)} - y = 6\sin x$.
4. Find a second order ODE whose solution is a family of circle with arbitrary radius and center on the x-axis, i.e., $(x-a)^2 + y^2 = b^2$ where a and b are arbitrary constants.

Fourier series, Fourier Transform and Partial differential equation

5. Write the Fourier series for $f(t) = |\cos t|$.
6. Find the Fourier series of a periodic signal with $f(t) = \exp(|-t|), \quad -1 < t < 1$
7. Find the (two – side) Fourier transform $F(W) = F\{f(t)\}$ of $f(t) = t\exp(-|t|)$.
8. Find the Fourier transform $X(f)$ of $x(t) = \exp(|-t|)\cos(2\pi f_c t)$.
9. Solve the partial differential equation $xz_x + z_t = xt$ for $z(x,t)$, $x \geq 0$, $t \geq 0$ with the condition $z(x,0) = 0$ and $z(0,t) = 0$ Hint : Use Laplace transform
10. Solve for $z(x,t)$ the partial differential equation

    $$z_{xx} = z_t + z, \quad t \geq 0, \ 0 \leq x \leq 1$$

    with the conditions $z_x(0,t) = z_x(1,t) = 0$ for all t and $z(x,0) = 2\sin^2 \pi x$ for all x.

Laplace and Inverse Laplace Transform

11. Find the laplace transform of $f(t) = e^{-2t}|\sin t|u(t)$.
12. Find the inverse laplace transform of $F(S) = \dfrac{1-2e^{-2s}+e^{-4s}}{s^2}$.
13. Find the inverse laplace transform of $F(S) = \dfrac{4(s^2+2)}{(s+1)(s^2+1)^2}$.

Eigenvalue and Eigenvector

14. Find the eigenvalue and eigenvector of $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

Vector space, Basis, Dimensions

15. Find condition on a, b, c so that $(a,b,c) \in R^3$ belongs to the space generated by $u = (2,1,0)$, $v = (1,-1,2)$, and $w = (0,3,-4)$.

16. Let W be the subspace of R^4 generated by the vectors $(1,-2,5,-3)$, $(2,3,1,-4)$ and $(3,8,-3,-5)$
    a. Find a basis and the dimension of W.
    b. Extend the basis of W to a basis of the whole space R^4.
17. Let U and W be subspaces of R^5 such that
    U is spanned by $\{(1,3,-3,-1,-4),(1,4,-1,-2,-2),(2,9,0,-5,-2)\}$
    W is spanned by $\{(1,6,2,-2,3),(2,8,-1,-6,-5),(1,3,-1,-5,-6)\}$
    a. Find the basis of $(U \cap W)$.
    b. Find dim $(U+W)$ and dim $(U \cap W)$.

Residues

18. Evaluate $\int_0^{2\pi} \frac{\sin^2\theta}{5+3\cos\theta} d\theta$.
19. Evaluate $\oint_c \frac{1}{z^3\left(z^2+2z+2\right)} dz$ where c is the counter – clockwise.
20. Evaluate $\int_{-\infty}^{\infty} \frac{2z^2-1}{z^3-z^2-4z-6} dz$.

System of linear equation

21. Find the value of k so that the solution of the following equations exists. By using that value of k, solve those equations.

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 3 \\ -4x_1 + x_2 + 7x_3 &= -5 \\ -2x_1 - 3x_2 + 11x_3 &= k \end{aligned}$$