

Let  $(X, \mathcal{B}, \mu)$  be a complete, finite measure space. For each sets  $A, B \in \mathcal{B}$ , define

$$d(A, B) = \mu(A \Delta B)$$

where  $A \Delta B$  is the symmetric different of  $A$  and  $B$ :

$$A \Delta B = (A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$$

Introduce the equivalence relation on the family of all measurable sets by declaring  $A$  and  $B$  equivalent if  $d(A, B) = 0$ . Let  $E$  be the space of equivalence classes. Prove that  $d$  introduces a metric on  $E$  and that  $(E, d)$  is a complete metric space.