Defining $\phi : \mathbb{Z} \to \mathbb{Z}_m \oplus \mathbb{Z}_n$ by $\phi(x) = \left([x]_m, [x]_n \right)$. Find the $\ker \phi$ and its image. Show that ϕ is onto iff $\gcd(m, n) = 1$.

Where the direct sum $\mathbb{Z} \oplus \mathbb{Z} = \{ (a_1, a_2) \mid a_1, a_2 \in \mathbb{Z} \}$ and addition and multiplication are componentwise.