4. Kirchhoff's voltage law says that the sum of the voltage drops around any closed path in the network in a given direction is zero. When this principle is applied to the circuit shown in Figure 3.5, we obtain the following linear system of equations:

$$
\begin{array}{rlrl}
\left(R_{1}+R_{3}+R_{4}\right) I_{1}+ & R_{3} I_{2}+ & R_{4} I_{3} & =E_{1} \\
R_{3} I_{1}+\left(R_{2}+R_{3}+R_{5}\right) I_{2}- & R_{5} I_{3} & =E_{2}  \tag{1}\\
R_{4} I_{1}- & R_{5} I_{2}+\left(R_{4}+R_{5}+R_{6}\right) I_{3} & =0
\end{array}
$$

Use Program 3.3 to solve for the current $\boldsymbol{I}_{1}, \boldsymbol{I}_{2}$, and $\boldsymbol{I}_{3}$ if
(a) $\boldsymbol{R}_{1}=1, \boldsymbol{R}_{2}=1, \boldsymbol{R}_{3}=2, \boldsymbol{R}_{4}=1, \boldsymbol{R}_{5}=2, \boldsymbol{R}_{6}=4$, and $\boldsymbol{E}_{1}=23, \boldsymbol{E}_{2}=29$
(b) $\boldsymbol{R}_{1}=1, \boldsymbol{R}_{2}=0.75, \boldsymbol{R}_{3}=1, \boldsymbol{R}_{4}=2, \boldsymbol{R}_{5}=1, \boldsymbol{R}_{6}=4$, and $\boldsymbol{E}_{1}=12$, $\boldsymbol{E}_{2}=21.5$
(c) $\boldsymbol{R}_{1}=1, \boldsymbol{R}_{2}=2, \boldsymbol{R}_{3}=4, \boldsymbol{R}_{4}=3, \boldsymbol{R}_{5}=1, \boldsymbol{R}_{6}=5$, and $\boldsymbol{E}_{1}=41, \boldsymbol{E}_{2}=38$

## (The problems are from Triangular Factorization.)

## Figure 3.5:



Figure 3.5 The electrical network for Exercise 4.

Program 3.3:

Program 3.3 ( $\boldsymbol{P A}=\boldsymbol{L} \boldsymbol{U}$ : Factorization with Pivoting). To construct the solution to the linear system $\boldsymbol{A} \boldsymbol{X}=\boldsymbol{B}$, where $\boldsymbol{A}$ is a nonsingular matrix.

```
function X = lufact(A,B)
%Input - A is an N x N matrix
% - B is an N x 1 matrix
%Output - X is an N x 1 matrix containing the solution to AX = B.
%Initialize X, Y, the temporary storage matrix C, and the row
% permutation information matrix R
    [N,N]=size(A);
    X=zeros(N,1);
    Y=zeros(N,1);
    C=zeros(1,N);
    R=1:N;
for p=1:N-1
%Find the pivot row for column p
    [max1,j]=max(abs}(A(p:N,p)))
%Interchange row p and j
    C=A(p,:);
    A(p,:)=A(j+p-1,:);
    A(j+p-1,: )=C;
    d=R(p);
    R(p)=R(j+p-1);
    R(j+p-1)=d;
if A(p,p)==0
    'A is singular. No unique solution'
    break
end
%Calculate multiplier and place in subdiagonal portion of A
    for k=p+1:N
        mult=A(k,p)/A(p,p);
        A(k,p) = mult;
        A(k,p+1:N)=A(k,p+1:N)-mult*A(p,p+1:N);
    end
end
%Solve for Y
Y(1) = B(R(1));
for k=2:N
    Y(k)= B(R(k))-A(k, 1:k-1)*Y(1:k-1);
end
%Solve for X
X(N)=Y(N)/A (N,N);
```

for $k=N-1:-1: 1$
$\mathrm{X}(\mathrm{k})=(\mathrm{Y}(\mathrm{k})-\mathrm{A}(\mathrm{k}, \mathrm{k}+1: \mathrm{N}) * \mathrm{X}(\mathrm{k}+1: \mathrm{N})) / \mathrm{A}(\mathrm{k}, \mathrm{k}) ;$
end

