

A Simple ring is an Artinian ring with no 2-sided ideals. Let K be a field. Let V be an n -dimensional vector space over K and let $A = \text{End}_K(V)$, the ring of $n \times n$ matrices over K . The ring is clearly Artinian since it has finite dimension over K . We want to see that the 2-sided ideal are either (0) or A . Let W be a subspace of V . Then $L_W = \{f \in A \mid f(W) = 0\}$ is clearly a left ideal of A . Let I be a left ideal, $W = \{x \in V \mid f(x) = 0 \text{ for all } f \in I\}$, so $I \subseteq L_W$. Let e_1, \dots, e_n be a basis of V with e_1, \dots, e_r a basis of W .

Show that if $f(x)$ has zero e_1 - coordinate for all $f \in I$, then $x \in W$. Note that if π permutes the e_i , then $\pi \circ f \in I$.