Let $D\_{8}$ denote the group of symmetries of the square. Denote by $a$ a rotation anticlockwise by ${π}/{2}$ about the centre of the square, and by $b$ a reflection through the midpoints of an opposite pair of edges.

1. Verify that each rotation in $D\_{8}$ can be expressed as $a^{i}$ and each reflection can be expressed as$ a^{i}b$, for$ i\in \left\{0,1,2,3\right\}$.
2. Verify the relations $a^{4}=e, b^{2}=e$ and$ b^{-1}ab=a^{-1}$. Explain how these relations may be used to write any product of elements in $D\_{8}$ in the form given in (i) above. Illustrate this with the example$ a^{3}ba^{2}b$.
3. Find the conjugacy classes of$ D\_{8}$.
4. Show that the rotations in $D\_{8}$ form a normal subgroup, $H$. Write down the distinct cosets$ Hg$. Compute the multiplication table of the quotient group${ D\_{8}}/{H}$. To which well-known group is ${G}/{H}$ isomorphic? Is the subgroup generated by $b$ normal in$ D\_{8}$?
5. Viewing the square in the real plane, centred at the origin, write down the $2×2$ matrix $ρ\left(a\right)$ which represents the rotation$ a$ and the$ 2×2$ matrix $ρ\left(b\right)$ which represents the reflection$ b$. Check that
$$ρ\left(a\right)^{4}=I\_{2}$$$$ρ\left(b\right)^{2}=I\_{2}$$$$ρ\left(b\right)^{-1}ρ\left(a\right)ρ\left(b\right)=ρ\left(a\right)^{-1}.$$

(This shows that you can define a homomorphism $ρ:D\_{8}\rightarrow GL\left(2,R\right)$ by letting$ ρ\left(a^{i}b^{j}\right)=ρ\left(a\right)^{i}ρ\left(b\right)^{j}$.)

1. By labelling the corners of the square or otherwise, write down the homomorphism $φ:D\_{8}\rightarrow S\_{4}$, verifying that
$$φ\left(a\right)^{4}=id$$$$φ\left(b\right)^{2}=id$$$$φ\left(b\right)^{-1}φ\left(a\right)φ\left(b\right)=φ\left(a\right)^{-1}.$$