A seismograph is a scientific instrument that is used to detect earthquakes. A simple model of a seismograph is shown below. It consists of a particle of mass m to which a pointer is attached. The particle is suspended by a spring of natural length l_0 and stiffness k and a damper of damping constant r from a platform of height d which is fixed to the Earth. Let the vertical displacements of the Earth, relative to a fixed origin O, be denoted by y and let the length of the spring and the damper be x, as shown in the following diagram.
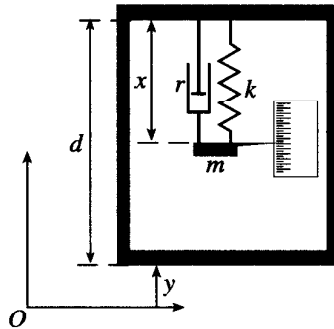
(a) Draw a force diagram showing all the forces acting on the particle.

(b) Express the forces acting on the particle in terms of the given variables and parameters.

(c) Show that the displacement $x(t)$ of the pointer with respect to the platform satisfies the differential equation

$$m\ddot{x} + r\dot{x} + kx = mg + kl_0 + m\ddot{y}.$$