(a) Prove the following theorem. Note that it is an "if and only if" theorem so you need to prove "if...then..." both ways.

Def: Let f be a function from X to Y. For $A \subseteq X$ and $C \subseteq Y$, then

$f(A) = \{ y \in Y : y=f(x) \text{ for some } x \text{ in } A \}$, and

$f^{-1}(C) = \{ x \in X : f(x) \in C \}$.

f(A) is called the **image of A,** and $f^{-1}(C)$ is called the **inverse image of C.**

Thm: Let f be a function from X to Y. f is onto if and only if $\forall\, C \subseteq Y,\ f(f^{-1}(C)) = C$

(b) Find an example of a function $f: X \rightarrow Y$ that is not onto and a subset of C of Y such that

$f(f^{-1}(C)) \neq C$