Express the 2nd order ODE

$$d\_{t}^{2}u=\frac{d^{2}u}{dt^{2}}=\sin(\left(u\right))+\cos(\left(ωt\right)) ω \in Z/\left\{0\right\}$$

$$u\left(0\right)=a$$

$$d\_{t}u\left(0\right)=b$$

as a system of 1st order ODEs and verify that there exists a global solution by invoking the global existence and uniqueness Theorem.

Useful information:

Global existence and uniqueness Theorem:

The ordinary differential equation
$$d\_{t}\overline{u}=\overline{f}(t,\overline{u}\left(t\right))$$

$$\overline{u}\left(0\right)=\overline{u}\_{0}$$

has a unique solution if $\overline{f}\in C^{0}(I)×Lipschitz(L\_{\infty }\left(R\right))$, f is continuous with respect to 1st variable and Lipschitz with respect to 2nd variable.

Lipschitz Continuity: A function $g:I\rightarrow R$ is Lipschitz continuous if $∃Λ>0$ such that
$\left‖g\left(\overline{x}\right)-g(\overline{y})\right‖\leq Λ\left‖\overline{x}-\overline{y}\right‖∀\overline{x},\overline{y}\in I$.

NB:$ \overline{}$ means vector value.