Given X is a general topological space:

Suppose $X = A_1 \cup A_2 \cup \ldots$, where $A_n \subseteq \mathring{A}_{n+1}$ for each n. If $f : X \to Y$ is a function such that, for each n, $f|A_n : A_n \to Y$ is continuous with respect to the induced topology on A_n, show that $f: X \to Y$ itself is continuous.

Please note that $\mathring{A}_{n+1}$ denotes the *interior* of the set A_{n+1}. So each A_n is embedded in the interior of the subsequent set.

Also, the union given above is an arbitrary union of A_i.

The notation $f | A_n : A_n \to Y$ denotes the restriction of the function f to A_n.