Section B

4. Find the Fourier series of the periodic function $f(x +2) = f(x)$ for all x, where

$$f(x) = \begin{cases} x & -\dfrac{1}{2} < x < \dfrac{1}{2} \\[2mm] 1-x & \dfrac{1}{2} < x < \dfrac{3}{2} \end{cases}$$

$$a_0 = 0$$

$$a_n = \frac{1}{n\pi}\left[-\frac{1}{2}\sin\frac{3n\pi}{2} - \frac{1}{2}\sin\frac{n\pi}{2} \right]$$

$$b_n = \frac{2}{n\pi}\sin\frac{n\pi}{2}\left(1-(-1)^n\right)$$

Ans. $f(x) = \dfrac{1}{\pi}\displaystyle\sum_{n\geq 1} \dfrac{1}{n}\left(-\dfrac{1}{2}\sin\dfrac{3n\pi}{2} - \dfrac{1}{2}\sin\dfrac{n\pi}{2}\right)\cos n\pi x + \dfrac{2}{\pi}\displaystyle\sum_{n\geq 1}\dfrac{1}{n}\sin\dfrac{n\pi}{2}\left(1-(-1)^n\right)\sin n\pi x$