

2. Suppose that we have the heat equation with the boundary-initial data

$$u_{xx} = k^{-1}u_t \quad 0 < x < a, \quad 0 < t$$

$$u(0,t) = T_0 \quad 0 < t, \quad u_t(a,t) = T_1 - u(a,t) \quad 0 < t, \quad u(x,0) = f(x) \quad 0 < x < a$$

where T_0 and T_1 are positive constants. Find a steady state solution of this equation. Use this knowledge to rewrite the solution $u(x,t)$ of the initial-boundary value problem in the form $u=v+w$ where $w(x,t)$ has homogeneous boundary conditions. Write down the initial-boundary value data problems that w and v satisfy.

Ans.

Steady state sol. $V_{xx} = 0 \quad V(0) = T_0 \quad V(a) = T_1$

$$V = \frac{1}{a} (T_1 - T_0)x + T_0$$