1. Find the displacement $u(x,t)$ of a string, given:

$$u_{tt} = 4\,u_{xx} \quad , \quad 0 \le x \le 2 \quad , \quad 0 \le t$$

$$u_x(0,t) = 0 \quad , \quad u_x(2,t) = 0$$

$$u(x,0) = \sin(\pi x) \quad , \quad u_t(x,0) = 3$$

2. The series of functions below (where it converges) defines the function $f(x)$:

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{x^2 + n^2} \cos(n\,x^n)$$

   (a) For what values of x does the series converge? Justify your answer.

   (b) Does the series converge *uniformly* where it converges? Justify your answer.

   (c) Is $f(x)$ continuous? Justify your answer.

   (d) Is $f(x)$ a periodic function? If so, what is the period?

   (e) Is $f(x)$ even, odd, or neither?

3. Given the two functions

$$h(t) = \begin{cases} t, & -4 \le t \le 0 \\ 0, & t < -4 \ \&\ t > 0 \end{cases}$$

$$g(t) = \begin{cases} e^{-2t}, & 2 \le t \\ 0, & t < 2 \end{cases}$$

   (a) Sketch $h(t)$ and $g(t)$.

   (b) Compute (from the definition) the convolution: $(h*g)(t)$. [Hint: carefully consider the different cases; sketches for various t may help.]

   (c) Find $H(f)$, the Fourier Transform of $h(t)$, by using the *definition* of the Fourier Transform (i.e., *evaluate the integral*).

   (d) Find $G(f)$, the Fourier Transform of $g(t)$, by using the Fourier *tables and properties.*

   (e) Using the convolution theorem and your previous answers, what is the Fourier Transform of the convolution: $\mathcal{F}\{(h*g)(t)\}$?

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