Case Studies on Real Options

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■ The introduction of option pricing theory (OPT) has been well received by practitioners, who have struggled with discounted cash flow (DCF) analysis for many years.[1] The ability of option pricing theory to quantify flexibility in strategic investment projects makes it a very appealing choice. This is especially so when one considers the fact that flexibility is often not explicitly taken into account by standard DCF analysis. Incorporating the value of flexibility could increase the total value of the project and may increase the probability of acceptance, an incentive for practitioners to try OPT in capital budgeting. The value of flexibility of an investment project is basically a collection of real options, which can be priced with the techniques known from financial options.

Despite this incentive, the process of adapting OPT to the practice of strategic decision-making is far from smooth. In most cases, the introduction of OPT would require practitioners to fundamentally reconsider their standard capital budgeting techniques. And when all this hard work has been done, there is still the question: how do we tell management? That question seems to lead us back to where we started. For practical purposes, we cannot afford to come up with very complicated options techniques that can only be priced with black-box computer programs.[2] The contribution of real options in practice is limited when one cannot explain what the important options are and why DCF analysis cannot be used.

In this paper, the major insights gained from practical case applications, developed in cooperation with the group planning and manufacturing functions in Shell's central offices and a number of Shell operating companies, are presented.[3] Shell's main interest was to conduct a number of exploratory studies on the use of OPT in capital budget-

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[1]See, for example, Kester [9], Myers [14], and Trigeorgis and Mason [19].

[2]See Triantis and Hodder [18].

[3]Permission was given by Shell to present these cases. For other real option applications, see, for example, Brennan and Schwartz [3], Siegel, Smith, and Paddock [16], and Trigeorgis [21].

ing decisions. The studies are part of a group planning program to adapt existing and develop new techniques for strategic decision-making. In practice, real world cases have to be simplified in order to keep the analysis tractable. This applies for the OPT as well as for a standard DCF analysis. During an initial round of discussions, a number of investment opportunities were selected which were of particular interest to Shell and appeared to be illustrative of the potential benefit of applying OPT in capital budgeting decisions. Three of these cases are described here in some detail.

Each case follows the same format. It begins with a problem and model description, followed by the presentation of the data and results. The depth of analysis differs from case to case. Sometimes more attention is paid to the model description, and sometimes more time is spent on data estimation. Since all cases are confidential, the data have been disguised and some details have been omitted, without changing the basic option-like characteristics of the decision problem. The paper is organized as follows: Section I analyzes the timing option in an offshore project. In Section II, a so-called growth or sequential option case is presented, where the introduction of OPT helped management to reformulate their investment proposal. Section III contains an abandonment decision of a refinery production unit. In Section IV, the important steps of the decision-making process, when options are involved, are presented. This is followed by a summary of the major insights gained by the practitioners involved. Finally, the conclusion discusses the major contributions of real options in practice.

I. Timing Option

This first case deals with a decision problem that is typical for the offshore oil and gas industry. It has been simplified substantially, because, as described here, it turned out to be a nice example to illustrate the difference between OPT and DCF analysis.

In order to explore and develop an oil field, companies can buy licenses from the government. In the exploration phase, they try to estimate the amount and quantity of the oil and gas reserve within that field. Data necessary to perform the estimation are obtained from drilling holes. The license for exploration typically expires after a certain time. When the exploration time has expired, the oil company has three possible strategies, assuming that no additional information is required to obtain more accurate estimates of the volume of the reserve. It can decide:

(*i*) Not to develop and thus to return the field to the government;

(*ii*) To start and develop the reserve immediately; or

(*iii*) To postpone development and thus extend the exploration phase.

In order to extend the exploration phase and hold onto the license, it is necessary for the company to drill a number of extra holes at some cost. It is assumed that these holes do not provide new information for the estimation of the size of the reserve. Consequently, the only potential benefit from drilling the holes is postponing the investment for a couple of years. It is further assumed that once the alternative to extend the exploration phase has been chosen, it is only possible to start development after the expiration of the extended exploration phase.

Under standard analysis, the company should choose the alternative with the highest net present value (NPV). The first and the second alternatives do not contain any real options and can therefore be evaluated by applying standard DCF analysis. Given the estimation of reserve size, a positive NPV results in project initiation; otherwise, the field would be returned. Initially, under the assumption that no new information on the size would occur, the third alternative was not taken seriously at all. Management wondered why anyone would be willing to incur extra costs without some concrete gain. Management did not readily recognize that deferring the investment could lead to a higher NPV in the future due to a potential increase in oil prices. Management had to be convinced that alternative three can be attractive because it provides an opportunity to postpone and therefore to wait for higher prices in the future.

Of course, the value of investing today has to be compared with the value of investing after a number of years. If the company were to accept the project now, it would forego the option to postpone. If it were to decide to postpone the investment, it would forego the net cash inflows that it could have received from the project in the meantime. If the option of waiting to invest is worth more than the additional costs, it may in fact be worthwhile to extend the exploration phase and wait for higher oil prices. Moreover, even when the NPV is negative but the option to wait net of the additional costs offsets this negative NPV, the project should also not be abandoned.

A naive application of DCF analysis assumes that the project will have to start immediately, irrespective of its future NPV. This does not recognize the opportunity for management to decide not to start the project at the end of the extended exploration phase if it is not desirable to do so. There is, of course, no obligation to start the development of the oil field, but only a right which will only be exercised if the future NPV is positive.

At this point, practitioners often come up with decision-tree analysis as an alternative. In my experience, it was necessary to point out what additional implicit assumptions are being made. Although it is well known from financial markets that the risk of an option changes over time, and with changing prices, we found it necessary to explain to management that this has consequences for the discount rates used in the decision-tree analysis. Still, when trying to determine the options embedded in the investment project, it is often convenient to use the decision-tree analysis as a basic framework.

The option to postpone the investment can be modelled more appropriately as follows. When the company buys the option to wait by incurring the costs of extra drilling, it buys the right to start development at the expiration date of the extended license, T. The benefit of exercising the option at the expiration date is the market value of the developed project, $V(T)$, and the cost is equal to the (single) investment outlay, K. In this case, the option to wait is similar to a European call option on an installed project with maturity date T. To simplify matters, it is assumed that the investment outlay is constant and irreversible in the sense that the installation can only be used for this project. It would not be difficult to extend the model for a stochastic investment outlay.[4] We subsequently define $W(V, t)$ as the value of the total investment project, which can be seen as the ownership right to an undeveloped project.

The standard OPT typically assumes that the underlying stock follows a geometric Brownian motion with a constant rate of return and a constant volatility. If OPT is applied to capital budgeting, the similarity between the underlying stock price and the present market value of a claim to the developed project is used. The total expected rate of return of the project is equal to the capital gains plus the pay-out rate on the project.[5] The pay-out rate on the project represents the opportunity cost of delaying completion of the project, or the expected net cash flow accruing from a producing project. The greater the pay-out rate on the project, the greater is the cost of holding the option.

In this offshore case, the risk of the net cash inflows of the developed project is assumed to be determined only by the natural resource prices (gas and oil); the investment outlay is assumed to be fixed. The resulting option on the cash flows of the developed field is thus analogous to a financial option on a dividend-paying stock,[6] whose value can be calculated from Merton's [13] formula:

$$W(V, \tau) = Ve^{-\delta\tau}N(h) - Ke^{-r\tau}N(h - \sigma\sqrt{\tau}), \tag{1}$$

where

V = present value of the developed reserve.
K = present value of the investment outlay.
δ = pay-out rate on the project.
τ = time to maturity $(= T - t)$.
σ = volatility of the logarithmic rate of return of V.
r = riskless interest rate.
$N(\cdot)$ = univariate normal distribution function.

In this case, we do not go into detail on obtaining the necessary data. Our goal here is merely to illustrate the impact of real options on capital budgeting decisions and to show that these decisions can significantly change if the value of the option to wait is properly taken into account.

The data in this case are determined as follows. The present value of the developed reserve, $V(t)$, is assumed equal to the value of the investment outlay K, or $V(t) - K$ is equal to zero. This implies that, if there is no development lag, the NPV of developing the field immediately, $V(t) - K$, is equal to zero. The time to maturity is set equal to the expiration time of the license, which in this case was two years. Determining the time to maturity of the option to wait is not always so clear-cut. Theoretically, the time to maturity could be very long, but in practice it is often determined by the time it takes for competitors to enter the market.

The interest rate of a riskless bond with a maturity of two years was five percent (in real terms). The volatility and the pay-out rate are difficult to estimate. (In the next case, the estimation of oil price volatility is discussed in more detail.) Here, volatility is estimated at 20% (as a base case), and for sensitivity purposes it is varied between ten percent and 30% annually. The pay-out rate can be determined from the estimated cash flows of the developed field. However, if the project has a finite time to maturity the pay-out rate will not be constant, as assumed by the above model. Siegel, Smith, and Paddock [16] use a number of 4.1% as pay-out rate for a case of offshore petroleum

[4]See McDonald and Siegel [12].

[5]The total expected return is the equilibrium rate established by the capital market on a comparable-risk financial asset and it includes an appropriate risk premium. This is the discount rate for determining the present value of a developed project.

[6]For example, see Brennan and Schwartz [3].

Exhibit 1. Total Investment Opportunity Value as a Percentage of Investment Outlay (K)

	Pay-Out Rate (δ)		
Volatility (σ)	0%	5%	10%
10%	9.6%	3.1%	-0.3%
20%	14.3%	8.2%	4.0%
30%	19.4%	13.3%	8.6%

Note: $V = K$; $\tau = 2$ years; $r = 5\%$ annually.

Exhibit 2. Total Investment Opportunity Value as a Percentage of Investment Outlay (K)

	Pay-Out Rate (δ)		
Volatility (σ)	0%	5%	10%
10%	2.9%	-0.4%	-1.6%
20%	8.0%	3.9%	1.2%
30%	13.1%	8.4%	5.0%

Note: $V = 0.9 \times K$; $\tau = 2$ years; $r = 5\%$ annually.

leases. A five percent base case pay-out rate is assumed here, and for sensitivity purposes it is varied between zero and ten percent annually.[7] The cost of drilling additional holes is set equal to two percent of the investment outlay. In Exhibit 1, the total value of the investment opportunity, net of the cost of drilling holes, is presented as a percentage of the investment outlay.

For example, for a base case of $\sigma = 20\%$ and $\delta = 5\%$, net investment opportunity value amounts to about eight percent of the investment outlay. Of course, when volatility increases, the value of the investment opportunity also increases. When the pay-out rate of the project is large, the option to wait to invest is low. When the volatility is low and the pay-out rate is high, the drilling costs are not justified by the value of the option to wait.

Even when the market value of the project is less than the investment outlay, it may still be profitable to extend the exploration phase if the option to wait is sufficiently large. In Exhibit 2, the same numbers are presented assuming that the present value of the completed project's net cash inflows, $V(t)$, is only 90% of the investment outlay K. The rest of the data remain the same. When the investment opportunity (the option to wait minus the two percent costs of drilling) is worth more than ten percent of the investment outlay, the decision to wait is justified. If the volatility is high enough and the pay-out rate is low enough, it is worthwhile to wait. Therefore, it may pay to incur additional costs in order to hold onto a temporarily unprofitable (but risky) project.

II. Growth Option

The growth option considered in this section is a pioneer venture, which is typically a project with a high investment outlay and relatively low net cash inflows. It is a manufacturing project with substantial investment costs necessary to prove technology in a period when the project on its own does not appear attractive. But when economic conditions improve, it is important to have the technology proven in order to maintain and enhance market position. Therefore, from a strategic point of view, the pioneer venture may make sense.

From a traditional cash flow perspective, i.e., DCF analysis, however, the pioneer venture was not profitable on its own, where this project was considered on a stand-alone basis. The strategic value of the project derived from the fact that investing in the pioneer venture provided management with the opportunity to invest in future commercial ventures (see Brealey and Myers [2]). Production of the commercial venture could be approximately four to five times the size of the pioneer venture production.

Before reformulating the investment problem as stated below, we discussed the options embedded in the pioneer venture when considered as a stand-alone project. Management had actually tried to incorporate the value of the option to wait assuming that it takes time to build the project (see Majd and Pindyck [11]). Management understood that every time an investment outlay has to be made, it can decide to continue, to wait, or to stop the project. This is similar to a sequential investment problem, where the value of the total investment project consists of the values of a series of call options on the market value of the installed project.[8] In view of the fact that it would take about four years to build the project, management tried to determine the total option value using the accompanying software of Majd and Pindyck [11]. As it turned out, the calculated total option value was still insufficient to justify the investment in the pioneer venture. This further con-

[7]Strictly speaking, the values the volatility can take on are related to the values of the pay-out rate. This has been ignored here.

[8]These options are, in fact, options on options, or compound options (see Geske [5]).

vinced management to regard the pioneer venture as an opportunity for growth instead of as a stand-alone project. In subsequent discussions, it was clarified that, due to technological reasons, there was only one time during the four years that it was possible to wait or to call off the project. In what follows, I first consider the simple base case in which no option during the building of the project is considered. Subsequently, I consider the one time at which it was possible to wait or to cancel the project.

In option pricing terms, "buying" the pioneer venture would give management the right to acquire a commercial venture by paying its investment outlay. The option will only be exercised if the commercial venture is profitable at the maturity date of the option. Investing in the pioneer venture today is thus similar to investing in a growth option. In a sense, the negative NPV of the pioneer venture is part of the cost of buying this growth option. The investment problem can therefore be restated as follows: does the value of the growth option justify the cost of buying this option? Reformulating the investment problem in this way was seen by management as a major contribution of OPT to capital budgeting.

In Exhibit 3, the time profile of the decision situation is presented. At present, management must decide whether to continue or not with the pioneer venture.[9] Once the decision has been made, it would take four years to build the pioneer venture. Because competition was expected to be strong, the maturity date of the option was set equal to the earliest possible time that (from a technological point of view) building of the first commercial venture could start. As mentioned earlier, the estimated lead on competitors is an important factor in determining the time to maturity of the option. Here, this was estimated to be in year seven, after the first three years of production of the pioneer venture. The building of a follow-up commercial venture would take another four years, ready to start production in year 11. The planning situation is illustrated in Exhibit 3.

Given this time schedule, the project can be formulated in option pricing terms as follows. The pioneer venture can be naively seen as a European call option on a futures contract, where the futures price, F, is equal to the value of the commercial venture in seven years. The exercise price is equal to the investment outlay in year seven. The time to maturity is equal to seven years. The nature of this

[9]The project was actually already started three years before, but the major investment outlay still had to be made during the coming year.

Exhibit 3. Planning Situation

First Stage		Second Stage	
Year 0	Year 4	Year 7	Year 11
Go ahead with pioneer venture or stop	Start-up production of pioneer venture	Decision moment to start commercial venture	Start-up production of commercial venture

standard European call option on a futures contract is given by the following equation (Black [1]):

$$W(V, \tau) = Fe^{-r\tau}N(h) - Ke^{-r\tau}N(h - \sigma\sqrt{\tau}), \qquad (2)$$

where

$$h = \frac{\ln(F/K) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}.$$

$N(\cdot)$ = univariate normal distribution function.

The risk from the project's net cash inflows comes from the relative (margin) risk between the input and the output prices of both ventures. Since input prices are less sensitive to crude oil price changes than the output prices, it is assumed that most of the risk of this project comes from uncertainties involving the output price (i.e., from uncertainties in the crude oil price).

As already noted, one complication in this case is the opportunity by management to either continue or stop the investment at some specific moment during the building of the project (i.e., after one year). This implies that instead of deciding to start and finish the whole pioneer venture now, management actually only has to decide to continue with the next phase. In the first phase, starting now and ending after one year, only the investment outlay needed during that year is involved; this, in return, provides management with an option to continue with the pioneer venture (including the option on the commercial venture). At the end of this phase, management can decide to exercise the first option, which implies completion of the pioneer venture. If the option is left to expire unexercised, management basically aborts the entire investment opportunity. The decision to exercise the option depends on the remaining value of the pioneer venture, which itself is an option on the commercial venture. Thus, the decision to continue with the pioneer venture after the first year also depends on the value of the commercial venture.

The first call option, with a time to maturity of τ^*, is written on the value of the pioneer venture, which in turn depends on the value of the commercial venture. The cost of exercising this option equals the remaining (negative) NPV of the pioneer venture, defined as K^*. Exercising this first option provides a second call option with time to maturity equal to $\tau - \tau^*$. The first option is, in fact, an option on an option (a compound option), since the completion of the pioneer venture provides another option.[10] In the compound option formulation, the flexibility of multistage decision-making is explicitly taken into account, increasing the value of the option. The following formula for a compound option (see Geske [5]) can be used:

$$W(V, \tau) = Fe^{-r\tau}M(k, h; \rho) - Ke^{-r\tau}M(k - \sigma\sqrt{\tau^*}, h - \sigma\sqrt{\tau}; \rho) - K^*e^{-r\tau^*}N(k - \sigma\sqrt{\tau^*}), \quad (3)$$

where

$$h = \frac{\ln(F/K) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}.$$

$$k = \frac{\ln(F/F_c) + \frac{1}{2}\sigma^2\tau^*}{\sigma\sqrt{\tau^*}}.$$

$N(\cdot)$ = univariate normal distribution function.

$M(a, b; \rho)$ = bivariate normal distribution function with a and b as upper and lower integral limits, and correlation coefficient ρ.

$$\rho = \left(\frac{\tau^*}{\tau}\right)^{1/2}.$$

F_c is the critical value of the project above which the first call option will be exercised.[11] Estimates for the values of the following input variables are thus needed:

F = present value of the cash inflows of the commercial venture as of year seven.

σ = volatility of the rate of change of the commercial venture.

K = present value of the capital expenditures of the commercial venture as of year seven.

K^* = present value of first year capital expenditure of the pioneer venture.

r = riskless rate of interest.

τ = time to maturity of the simple option.

τ^* = maturity date of the first option (within the compound option).

It is relatively simple to determine the last three variables. Because investors are interested in the after-tax rate of return, the appropriate riskless rate is the after-tax real return on treasury securities. This rate, based on U.S. \$, was equal to two percent annually (in real terms). The time to maturity of the simple option is equal to seven years. The compound option has a time to maturity of one year and six years, respectively, which implies that τ^* is equal to one year. For all the other variables, market or project data are required. For both the pioneer and the commercial ventures, the production cash flows and the capital expenditures (capex) were estimated. Since I was not allowed to give the discount rate used by Shell, I disguised the data by presenting them as percentages in Exhibit 4. The percentages for the capital expenditures add up to 100%. The percentages for the production cash flows are percentages of the total production per year.

Two items should be noted. The three percent capital expenditures of the pioneer venture are treated as sunk costs and will not be taken into account. The technological uncertainty of the project is only reflected in the lower production level for the first three years for both the pioneer and the commercial venture.[12] Since management has a claim on the future net cash inflows of the commercial venture, the present value of the commercial venture as of year seven is calculated. This value should be regarded as an estimate for the market price of a futures contract for delivery of the net cash inflows of the commercial venture in year seven. It is assumed that this present value is already adjusted for the foregone pay-out rate during the time to maturity of the option. The fact that it takes time to build the project, of course, has consequences for the calculation of the present value of a producing venture, because the cash inflows arrive four years later. This is incorporated by calculating the present value of the commercial venture as of year 11, and discounting this value back to year seven. This results in a future value of U.S. \$1000 million as of year seven.

Since the volatility is not observable, it must be estimated empirically. Again, the main source of uncertainty

[10]See Geske [5].

[11]This value can be obtained using a simple Newton-Raphson procedure.

[12]There are different alternatives to deal with the technological uncertainties. One possibility is to incorporate uncertain capital expenditures (see, for example, Fisher [3]). However, management appeared relatively certain about the estimation of the capital expenditure. Another possibility is to model the first years of production as compound options, which would increase complexity substantially.

Exhibit 4. Production Revenues and Capital Expenditure (Capex) Percentages

					Pioneer Venture					
Year	0	1	2	3	4	5	6	7	8 . . .	25
Production						25%	50%	75%	100% . . .	100%
Capex	3%	18%	41%	27%	11%					
					Commercial Venture					
Year	7	8	9	10	11	12	13	14 . . .	25	
Production					25%	50%	75%	100% . . .	100%	
Capex	20%	40%	25%	15%						

is the difference between the supply costs and the output proceeds of the project. Since the supply costs are relatively insensitive to crude oil prices compared to the output proceeds, the volatility of the rate of return in the crude oil price is used as an upper bound for the volatility of the rate of return of the PV of the commercial venture. One problem encountered when estimating this volatility measure is that only historical data are available. There are no long-term financial options on oil (yet) from which we can calculate the implied volatility. On the basis of historical data, it is clear that the volatility of oil is not stable over time. Depending on the time horizon taken (between two and ten years), oil volatility has fluctuated between 15 and 20% annually.[13] Based on forecasts by company executives, it was expected that volatility could even be as much as 25%. The results are shown for a volatility of 15%, 20%, and 25% annually.

The present value of investment outlays is derived by discounting the total capital expenditures at the after-tax real riskless rate of two percent.[14] This results in U.S. \$1000 million. The total expenditures are based on the expected efficiency improvement with respect to the capital expenditures compared with the pioneer venture. For the compound option formulation, the present value of the remaining (negative) NPV of the pioneer venture is also required. This was estimated at U.S. \$90 million. In short, the following values were used to determine the value of the options:[15]

F = U.S. \$1000 million.
σ = 15%, 20% and 25% annually.
K = U.S. \$1000 million.
K^* = U.S. \$90 million.
r = 2% annually.
τ = 7 years.
τ^* = 1 year.

To calculate the simple option, Equation (2) can be used. The value of the simple option must then be compared with the (negative) NPV of the pioneer venture. When the option value exceeds this negative NPV, the pioneer venture should be continued. To calculate the NPV of the pioneer venture, its cash outflows are discounted at a riskless rate of two percent (in real terms), and all net cash inflows at the company's cost of capital (in real terms). This resulted in a negative NPV of −300 + 100 = U.S. \$ −200 million. To calculate the compound option value, Equation (3) is used. The value of the compound option is then compared with the present value of the first-year capital expenditures of the pioneer venture. When the compound option value exceeds these costs, the pioneer venture should be continued. The present value of the first-year capital expenditures is estimated at U.S. \$98 million.

In Exhibit 5, the option value for various volatility estimates is presented. The first row contains the value of the simple option minus its costs. The second row contains the value of the compound option net of its costs. In the

[13] Pindyck [17] tests whether the oil price returns are mean-reverting or not, but could not find evidence that oil price returns are indeed mean-reverting. However, if mean reversion is assumed, this would lead to a decrease in the estimated option value.

[14] This ignores the option to wait during the building period of the commercial venture. However, as for the pioneer venture, this option to wait was considered irrelevant due to competition.

[15] Although the numbers presented here are not the exact numbers used in the real project, the relative order is the same.

Exhibit 5. Total Investment Value in Million U.S. $

	Volatility (σ)		
	15%	20%	25%
Simple option	-65	-19	25
Compound option	-41	0	43
Critical value F_c	812	730	653

third row, the critical value below which the project should be stopped after one year is also given.

The results indicate that, in case of a low volatility, the pioneer venture should not be started. However, based on the compound option value, the investment can be justified when volatility is 20% or higher. If management is able to modify its decisions more frequently, this will further increase the value of the total investment opportunity.

III. Abandonment Option

In this section, the abandonment decision of a crude distiller (CD) in a refinery is discussed. At some point, management had decided to abandon this crude distiller. One reason was that during the last couple of years the supply of distillates from crude oil exceeded the demand, and despite increased rationalization in production, there was still pressure on the price of distillates produced by the refining industry. This case was analyzed because it was felt that it might yield insights useful for future abandonment decisions in this industry.

Although not considered in this section, management was also interested in applying OPT to a more general strategic problem in the refinery industry. In this competitive industry with over-capacity, management has to continuously consider whether to stay in business or to get out. The advantage of staying in business is that when other competitors get out, the company can take over their market share and ultimately extend its business. The costs of staying in business are the losses that are incurred by maintaining production at a given level. Although this strategic problem is difficult to quantify, it helps if it is structured in terms of options. It was felt by management that thinking in terms of options would force them to analyze their competitive situation more explicitly. Basically, in this situation there is an option to abandon and an option to extend the business. Whatever management may decide, it would depend on the critical project value below which abandonment takes place and on the critical project value above which extension of production would take place.[16] However, since the abandonment decision of the CD itself was already complicated enough, the more general problem (with potential interactions) will not be presented here.

In principle, the CD produces a wide range of products, but due to the small resulting margin it was only used to bridge the shutdowns of other crude distillers. When it turned out that there existed another (cheaper) alternative to bridge the shutdowns, the justification of continued maintenance was undermined. The only reason to hold onto the CD would be a sufficient margin to justify the high upfront revision costs and the annual operating costs. Since, at the time, it was expected that the margin would be insufficient, the CD was abandoned.

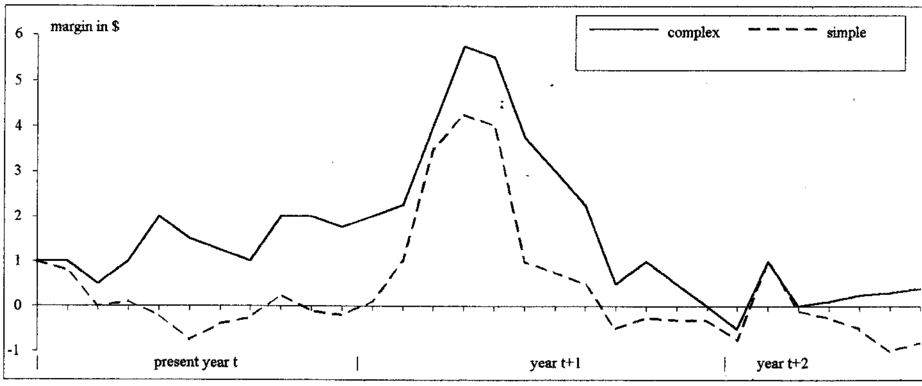
The major source of uncertainty in the cash flows of the CD was the difference in value between the supply cost (i.e., crude prices) and the output proceeds determined by the yields of different products (e.g., naphtha, gasoline, fuel oil) and their respective prices on the open market. For an installation of this type, this is referred to as a "simple margin." For a crude distiller coupled with a number of other installations that further upgrade the product package, it would be called a "complex margin." For the relevant time period, average monthly data of the complex and simple margins are presented in Exhibit 6.

During the two years prior to the decision, the simple margin was almost always negative. In the year of the decision, the simple margin recovered slightly and was above the acceptable level of U.S. $.40/barrel for about two or three months. A substantial increase in the simple margin above the U.S. $4.00/barrel followed in the first part of the subsequent year.

On the basis of the simple margin, it was unlikely that the operating costs would be covered. Certainly the upfront revision costs would not be justified. Even when the cash inflows would exactly offset the operating costs every year, the resulting NPV would be negative due to the upfront revision costs. In brief, the NPV of the project, based on the simple margin at the time of the decision, lead to the decision to abandon the project. However, due to the high margin in the following year, the NPV might have been positive. There are, of course, no guarantees to prevent margins from dropping again. If, in future years,

[16]It must be recognized that the combined value of these two options is not equal to the sum of separate values due to interactions; see Trigeorgis [22].

Exhibit 6. Average Monthly Data of the Complex and Simple Margins



simple margins could frequently be above the U.S. \$.40/barrel, it might be worthwhile to hold onto the CD.

The decision to abandon the CD can be seen as owning the project and having a put option to abandon at every relevant moment (here, in every year during the technical life of the CD).[17] The project is similar to an American put option with a limited amount of exercise possibilities. The benefit of abandonment now is equal to sum of the present value of the future operating costs, $K(t)$, and the upfront revision costs, *REVC*. The loss is equal to the sum of the present value of the future cash inflows, $V(t)$, and the opportunity to abandon the CD in the future.

Unfortunately, it is relatively complicated to solve the American put problem, and therefore numerical methods are needed to calculate its value. The approach applied here is based on the method of Geske and Johnson [6]. Let us assume that the technical life of the CD is equal to $T + 1$ years. After T years, management decides whether to stop or continue the CD on the basis of the remaining NPV of the CD. If the expected cash inflows, $V(T)$, are less than or equal to the production costs, $K(T)$, production is stopped. This can be seen as exercising a put option at the maturity date. When management decides to continue the project at year T, there is no option left since the project will be stopped at $T + 1$ with certainty. At year $T - 1$, management has the opportunity to again stop or to continue. It will decide to stop production if $V(T - 1)$ is below some critical value $V_c(T - 1)$. This critical value is the value for which management is indifferent between stopping or continuing. At year $T - 1$, the value of stopping is equal to $K(T - 1) - V(T - 1)$, and the value of continuing is equal to a European put option with time to maturity of one year and an exercise price equal to $K(T)$. This implies that $V_c(T - 1)$ can be found by solving the following equation:

$$K(T-1) - V_c(T-1) = P[V_c(T-1), K(T-1), 1], \qquad (4)$$

where $P(V, K, \tau)$ is equal to the value of a European put to give up V in exchange for K with maturity τ. If $V(T - 1)$ is below $V_c(T - 1)$, the project should be abandoned. However, at year $T - 1$, this possibility to abandon only exists when the project has not been abandoned on or before year $T - 1$. Going back another year, the decision to stop can be taken at year $T - 1$ and at year T, provided that the project has not been abandoned before. Management has a put option that can be exercised at year $T - 1$ or year T. The put option can be divided into two separate put option values. The first put option, expiring at year $T - 1$, gives the right on a second put option expiring at year T. The value of the second option is conditional on no early exercise at year $T - 1$. The put option value is given by (see Geske and Johnson [6]):

$$\begin{aligned} P[V, K(T-1), K(T), 2] = {} & K(T-1)e^{-r}N(-k+\sigma) - VN(-k) \\ & + K(T)e^{-2r}M(k-\sigma, -h+\sigma\sqrt{2}; -1/\sqrt{2}) \\ & - VM(k, -h; -1/\sqrt{2}), \end{aligned} \qquad (5)$$

where

[17]This abandonment decision can also be seen as a sequential investment decision. Every year management has the opportunity to exercise a call option, obtaining the remaining future cash inflows by paying the cash outflow in that year. The benefit of exercising the option is equal to the net cash inflows of that year plus the opportunity to continue the project in the future. The costs are equal to the cash outflows of that year (see Myers and Majd [15]).

$$h = \frac{\ln(V/K(T)) + 2(r + \tfrac{1}{2}\sigma^2)}{\sigma\sqrt{2}}.$$

$$k = \frac{\ln(V/V_C(T-1)) + (r + \tfrac{1}{2}\sigma^2)}{\sigma}.$$

$N(\cdot)$ = univariate normal distribution function.

$M(a, b; \rho)$ = bivariate normal distribution function with a and b as upper and lower integral limits, and correlation coefficient ρ.

Again, this put option will only have value when the project is not abandoned at year $T-2$. This will be the case when $V(T-2)$ is above the critical value $V_C(T-2)$. This analysis can be repeated for $T-2, T-3, T-4, \ldots$ up to the present year t. At the present time, the value of a put option that can be exercised every year is computed. This put value consists of T pairs of European put options, one for each year in which abandonment is possible. All the European put options are conditional on not being exercised before that year, which results in multinomial distribution functions for the American put value at time t. These multinomial distribution functions can be approximated by a log-transformed explicit finite difference method (e.g., see Geske and Shastri [7]). The following input variables are to be estimated:

$V(t)$ = the present value of the future cash inflows.
$K(t)$ = the present value of the future operating costs.
σ = the volatility of the rate of return on $V(t)$.
r = the riskless rate.
τ = the time to maturity.
$REVC$ = the revision costs.

The last three variables do not cause any problems. The time to maturity is equal to nine years, the after-tax riskless rate, based on guilders, is set equal to 6.5% annually. The revision costs are equal to the present value of the after-tax capital expenditures necessary to upgrade the production unit, which amounted to 6 million guilders. In order to determine $V(t)$ and $K(t)$, we have to determine the cash inflows and the operating costs of the production unit. Irrespective of the number of weeks of production, the after-tax operating costs are 4 million guilders per year, that is, in case the production unit is kept ready for production. As mentioned earlier, production will only take place when the simple margin exceeds U.S. \$.40/bbl. Based on the simple margin of the year in which the decision is taken (and taking into account the fact that it takes one week to start producing), it is estimated that the CD would produce for approximately eight weeks at a minimum simple margin of U.S. \$.40/bbl. Using a constant exchange rate of fl 2.00/\$, the yearly after-tax cash inflow would be equal to 4.8 million guilders per year.

This implies that, when the firm's after-tax cost of capital is used to discount the cash flows, the present value of the future cash inflows is equal to 30 million guilders and the present value of the future operating costs is equal to 25 million guilders. Using a standard DCF analysis, this project would be abandoned immediately since the NPV is equal to $-6 - 25 + 30 = -1$ million guilders. In order to show the sensitivity of the put option value with respect to $V(t)$, we also show the results when $V(t)$ equals 25 million (which results in an at-the-money put option) and 31 million guilders (which results in a zero NPV for the total investment opportunity).

The uncertainty of the cash inflows stems directly from the uncertainty in the simple margin. The evolution of $V(t)$ over time is determined by the development of the simple margin. In order to use the standard option pricing model, we need to adopt the assumption that the logarithmic rate of return of $V(t)$ is normally distributed with volatility σ. This volatility is equal to the volatility of the logarithmic rate of return of the simple margin assuming that the simple margin is log-normally distributed. This can be achieved by defining the simple margin as the percentage between the output proceeds and the supply costs and assuming that both are log-normally distributed. Using weekly data on the output proceeds and supply costs over the year preceding the decision moment, the volatility was estimated as low as 5.8%. However, in the subsequent year, volatility was estimated as high as 24.4%, after which it dropped to 11.3% in the following year. Therefore, the results are presented for a volatility between five and 20%. In all, the results are presented for the following input variables:

$V(t)$ = 25, 30, 31 million guilders.
$K(t)$ = 25 million guilders.
σ = 5, 10, 15 and 20%.
r = 6.5%.
τ = 9 years.
$REVC$ = 6 million guilders.

In Exhibit 7, the value of the put option for the various $V(t)$ and volatilities is given. In the last row, the critical value of $V(t)$ below which the project should be abandoned immediately (V_C) is presented. When the value of $V(t)$ plus the value of the put option exceeds $K(t)$ plus the revision costs (= 31 million guilders), the project should not be abandoned.

From these results it follows that for $V(t) = K(t) = 25$ million guilders, the project would always be abandoned.

Exhibit 7. Put Option Values

	Volatility			
	5%	10%	15%	20%
$V = 25$	0.06	0.53	1.27	2.16
$V = 30$	0.00	0.05	0.42	1.11
$V = 31$	0.00	0.03	0.34	0.98
V_C	31.00	30.97	30.64	29.88

For $V(t)$ equal to 30 million guilders, the project would not be abandoned when volatility is as high as 20%, because $V(t)$ exceeds V_C. Given the available information at the moment of the decision (a volatility of no more than ten percent), it appears that management made the right decision to abandon the project.

IV. Major Insights

From the experience gained from these and other cases developed in cooperation with Shell planning group, an attempt was made to develop a more general formulation of the decision-making process when such options are involved. This resulted in conceptualizing various steps that were considered important in the decision-making process, as follows:

(*i*) Convince management that some proposals contain flexibility that cannot be valued by using DCF analysis and must be valued using OPT.[18]

(*ii*) Make a clear distinction between investment alternatives and options embedded in these alternatives, because management often considers options as alternatives, which leads to misinterpretations.

(*iii*) Restrict the number of options to the most important ones; more options increase complexity without necessarily adding much value.

(*iv*) Restate the investment problem in the following sense: Can the costs of the (additional) flexibility be justified by the benefits when the flexible alternative is compared to the alternative without flexibility?

(*v*) Define properly the uncertainties that management faces and, given these uncertainties, determine the most valuable option(s).

(*vi*) Whenever possible, incorporate the influence of competitors and other costs that may affect the value of the option(s). Sometimes management takes the option(s) for free. It may be possible that due to the specific situation of the firm, the option is cheaper to the firm than to other firms. But options are usually not free. It is also important to incorporate the effects of competition not only on the cash flow estimates, but also on the value of the option (e.g., see Trigeorgis [20]).

(*vii*) Focus on the value of the project including the option(s) and present sensitivity analysis, especially for volatility.

When applying OPT in practice, it is important to come to a more general approach in the decision-making process, but it seems equally relevant to find out the opinion of the practitioners involved. This provides insight into how they position the new technique and how they cope with it. Some major insights gained by practitioners involved in these cases, are as follows:

(*i*) The same fundamental principles underlie both DCF analysis and OPT.

(*ii*) DCF is a simplified technique, which is appropriate for the analysis of a broad range of problems under passive management. When it is known that different elements of a cash flow are associated with different risks, this should be reflected by applying different discount rates.

(*iii*) DCF and OPT are complementary rather than competing techniques. OPT should be used in combination with DCF when there are future decision points which influence the riskiness of the cash flow.

(*iv*) OPT is rather like an appropriate combination of discounting and decision-tree analysis. It is particularly useful for analyzing the value and phasing of a series of related investments.

(*v*) DCF analysis has probably been sufficient for the evaluation of most traditional expansion projects. As normally applied, however, it systematically undervalues the benefits of waiting.

(*vi*) These techniques do not, of course, replace the need for strategic thinking and judgment in the generation and examination of business alternatives. When they are properly applied, however,

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[18] A recent paper by Kulatilaka and Marcus [10] focuses specifically on this issue.

they could be of invaluable support to this activity by enabling a meaningful quantification as part of the evaluation process. As one participant noted, "It's a shame to do all this hard strategic analysis and then throw it away at the arithmetic stage."

V. Conclusion

The theoretical foundations of the above cases were mostly limited to the more simple option models. But even these simple option models can provide management with considerable intuition. The above cases were developed with Shell staff members, who were unfamiliar with the theory. But the basic outcomes and the sensitivities of these outcomes to changes in the underlying input variables, seemed consistent with their intuition. When applying OPT, a major problem is to decide on the most important embedded options; what are they, and which of them are potentially valuable to model? In the cases that we looked at, most problems had to be simplified, although this was also common practice when a standard DCF analysis was used.

Based on these experiences, we suggest that the main contribution of OPT in capital budgeting is twofold. First, it helps management to structure the investment opportunity by defining the different investment alternatives with their underlying uncertainties and their embedded options. A side-benefit is that it usually leads to a renewed discussion about the use of standard capital budgeting techniques. Second, OPT can handle flexibilities within the project more easily than the traditional DCF analysis. Although other models such as decision-tree analysis or Monte Carlo simulation could be used, they tend to become complicated and are frequently misapplied.[19]

The application of OPT in practical capital budgeting decisions is not without problems either. When the real options in the investment projects tend to become more complex, the OPT approach also becomes complicated and computations become increasingly difficult. It is also not always easy to find a good estimate for the uncertainty of the underlying project. It is thus not surprising that in most real world cases OPT has been applied to investment projects whose cash flows are based on quoted natural resource prices. In conclusion, I suggest that further research on real options should focus on unanswered questions arising from practical investment problems.

[19]An interesting method, proposed by Jacoby and Laughton [8], tries to find a balance between the standard capital budgeting techniques and the more stochastic-oriented OPT.

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