Attachment 1

TOTAL PAGES = 2

Exhibit 11.15 Comparison of 3-Month Moving Average and Weighted Moving Average Models

	A	B	C	D	E	F	G	H	I
1	Gas-Mart Monthly Milk Sales							Error Analysis	
2	Month	Sales	3-Month MA	Error	3-Month Weighted	Error		3-Month MA	3-Month Weighted
3	1	172							
4	2	217							
5	3	190							
6	4	233	193.00	40.00	187.30	45.70		1600.00	2088.49
7	5	179	213.33	-34.33	210.50	-31.50		1178.78	992.25
8	6	162	200.67	-38.67	201.80	-39.80		1495.11	1584.04
9	7	204	191.33	12.67	209.70	-5.70		160.44	32.49
10	8	180	181.67	-1.67	176.40	3.60		2.78	12.96
11	9	225	182.00	43.00	176.40	48.60		1849.00	2361.96
12	10	250	203.00	47.00	198.90	51.10		2209.00	2611.21
13	11	151	218.33	-67.33	200.50	-49.50		4533.78	2450.25
14	12	218	208.67	9.33	225.10	-7.10		87.11	50.41
15								13116.00	12184.06
16							MSE	**1457.33**	**1353.78**

Single Exponential Smoothing

Single Exponential smoothing (SES) *is a forecasting technique that uses a weighted average of past time-series values to forecast the value of the time series in the next period.* SES forecasts are based on averages using and weighting the most recent actual demand more than older demand data. SES methods do not try to include trend or seasonal effects. The basic exponential-smoothing model is

$$F_{t+1} = \alpha A_t + (1 - \alpha)F_t = F_t + \alpha(A_t - F_t) \qquad \textbf{(11.6)}$$

Single Exponential smoothing (SES) *is a forecasting technique that uses a weighted average of past time-series values to forecast the value of the time series in the next period.*

where α is called the **smoothing constant** ($0 \leq \alpha \leq 1$). To use this model, set the forecast for period 1, F_1, equal to the actual observation for period 1, A_1. Note that F_2 will also have the same value.

Using the two preceding forms of the forecast equation, we can interpret the simple exponential smoothing model in two ways. In the first model shown in Equation 11.6, the forecast for the next period, F_{t+1}, is a weighted average of the forecast made for period t, F_t, and the actual observation in period t, A_t. The second form of the model in Equation 11.6, obtained by simply rearranging terms, states that the forecast for the next period, F_{t+1}, equals the forecast for the last period, F_t, plus a fraction, α, of the forecast error made in period t, $A_t - F_t$. Thus, to make a forecast once we have selected the smoothing constant, we need only know the previous forecast and the actual value.

To illustrate the exponential-smoothing approach to forecasting, consider the milk-sales time series presented in Exhibit 11.16 using $\alpha = 0.2$. As we have said, the exponential-smoothing forecast for period 2 is equal to the actual value of the time series in period 1. Thus, with $A_1 = 172$, we will set $F_1 = 172$ to get the computations started. Using Equation (11.6) for $t = 1$, we have

$$F_2 = 0.2A_1 + 0.8F_1 = 0.2(172) + 0.8(172) = 172.00$$

For period 3 we obtain

$$F_3 = 0.2A_2 + 0.8F_2 = 0.2(217) + 0.8(172) = 181.00$$

By continuing these calculations, we are able to determine the monthly forecast values and the corresponding forecast errors shown in Exhibit 11.16. The mean squared error is MSE = 1285.28. Note that we have not shown an exponential-smoothing forecast or the forecast error for period 1, because F_1 was set equal to A_1 to begin the smoothing computations. You could use this information to generate a forecast for month 13 as

$$F_{13} = 0.2A_{12} + 0.8F_{12} = 0.2(218) + 0.8(194.59) = 199.27$$

Exhibit 11.17 is the plot of the actual and the forecast time-series values. Note in particular how the forecasts "smooth out" the random fluctuations in the time series.

Exhibit 11.16
Summary of Single Exponential Smoothing Milk-Sales Forecasts with $\alpha = 0.2$

	A	B	C	D	E
1	Gas-Mart Monthly Milk Sales				
2		Alpha	0.2		
3	Month	Sales	Exponential Smoothing Forecast	Error	Error^2
4	1	172	172.00		
5	2	217	172.00	45.00	2025.00
6	3	190	181.00	9.00	81.00
7	4	233	182.80	50.20	2520.04
8	5	179	192.84	-13.84	191.55
9	6	162	190.07	-28.07	788.04
10	7	204	184.46	19.54	381.91
11	8	180	188.37	-8.37	69.99
12	9	225	186.69	38.31	1467.44
13	10	250	194.35	55.65	3096.44
14	11	151	205.48	-54.48	2968.44
15	12	218	194.59	23.41	548.18
16				MSE	1285.28

Exhibit 11.17
Graph of Single Exponential Smoothing Milk-Sales Forecasts with $\alpha = 0.2$