

10-3 A slab of material has parallel faces. One coincides with the xy plane while the other is given by $z = t$. The material has a nonuniform polarization $\mathbf{P} = P(1 + \alpha z)\hat{\mathbf{z}}$ where P and α are constants. Find the volume and surface densities of bound charge. Find the total bound charge in a cylinder of the material of cross section A and sides parallel to the z axis and thus verify directly that (10-13) holds for this case.

$$\begin{aligned}
 Q_b &= \int_{V'} \rho_b d\tau' + \oint_{S'} \sigma_b da' = - \int_{V'} \nabla' \cdot \mathbf{P} d\tau' + \oint_{S'} \sigma_b da' \\
 &= - \oint_{S'} \mathbf{P} \cdot \hat{\mathbf{n}}' da' + \oint_{S'} \mathbf{P} \cdot \hat{\mathbf{n}}' da' = 0
 \end{aligned}
 \tag{10-13}$$