Homework Five.

1) The current as a function of time that flows during a lightning strike usually has a 'zig-zag' appearance as a typical strike is made of 3 to 4 strokes. High speed videos (examined frame-by frame) show that most lightning strikes are made up of multiple individual strokes. However, imagine that the current varies smoothly as a function of time and can be described as

$$I(t) = k(t - at^2)$$

where $k = 1 \times 10^4 As^{-1}$, $a = 10s^{-1}$ and the time, t lasts for $100ms$. Find the total amount of charge which flows during the strike.

2) Determine and plot both the E-field and electric potential for both a charged conducting sphere and a uniformly charged insulating sphere.

3) A thin circular disk with a circular hole at its center (called an annulus) has inner radius R_1 and outer radius R_2. The disk has a uniform positive surface charge density σ on its surface. i) Determine the total electric charge on the annulus. ii) If the annulus lies in the y-z plane with its center at the origin find the magnitude and direction of the electric field along the x-axis. Consider points both above and below the annulus. iii) What would happen to a negative charge if it was allowed to move only along the x-axis and was placed at $x = 0.01R_1$?

4) A region in space contains a total positive charge Q that is distributed spherically such that the volume charge density $\rho(r)$ is given by

$$\rho(r) = \begin{cases} \frac{3\alpha r}{2R} & \text{if } r < R/2, \\ \alpha\left[1 - \left(\frac{r}{R}\right)^2\right] & \text{if } R/2 < r < R. \\ 0 & \text{if } r > R. \end{cases}$$

Here α is a positive constant having the units of Cm^{-3}. i) Determine α in terms of Q and R. ii) Using Gauss' law derive an expression for the magnitude of the electric field as a function of r.

5) The electric potential $V(r)$ due to a spherically symmetric nonuniform charge distribution is of the form

$$V(r) = \frac{\rho_0 a^2}{18\epsilon_0}\left[1 - 3\left(\frac{r}{a}\right)^2 + 2\left(\frac{r}{a}\right)^3\right]$$

for values of r less than a, for $r > a$ the electric potential is zero. ρ_0 is a constant with units Cm^{-3} and a is a constant with units of meters. i) Derive an expression for the electric field E_r in the radial direction. ii) Derive an expression for the charge density $\rho(r)$. iii) **Sketch your results.**

Bonus Question, 20%) Two thin rods of length L lie along the x-axis, one between $x = a/2$ and $x = a/2 + L$, and the other between $x = -a/2$ and $x = -a/2 - L$. Each rod has a positive charge Q distributed uniformly along its length. Show that the magnitude of the force that one rod exerts on the other is

$$F = \frac{Q^2}{4\pi\epsilon_0 L^2} \ln\left[\frac{(a+L)^2}{a(a+2L)}\right]$$