

1. Let Y be a topological space, Γ —a family of functions $f : X \rightarrow Y$. Assume X has a Γ -topology on it.
 - (a) Prove that $x_n \rightarrow x$ in X if and only if $f(x_n) \rightarrow f(x)$ in Y for all $f \in \Gamma$.
 - (b) Suppose Z is a topological space. Prove that $g : Z \rightarrow X$ is continuous if and only if $f \circ g : Z \rightarrow Y$ are continuous for all $f \in \Gamma$.
2. Let K be a closed and convex subset of a locally convex topological vector space X . Prove that K is also closed in X_w . Notice that if topology in X_w is strictly weaker than topology on X then there will be closed sets in X that are not closed in X_w . So, convexity plays a very important role.
3. Let X be a topological vector space. Prove that X_w^* is a locally convex topological vector space