

1) Which of the sets below are submanifolds of \mathbb{R}^n ? Of what differentiability class?

(1) $\{(x, y, z) \in \mathbb{R}^3 : x^3 + y^3 + z^3 - 3xyz = 1\}$.

(2) $\{(x, y, z) : x^2 + y^2 + z^2 = 1, x^2 + y^2 - x = 0\}$.

(3) $\{(t, t^2) : t \leq 0\} \cup \{(t, -t^2) : t > 0\}$.

2) Let (a_{ij}) be a symmetric, invertible matrix. Show that

$$S = \{x \in \mathbb{R}^n : \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j = 1\}$$

is a submanifold of \mathbb{R}^n . What is its dimension?