The displacement u(x, t) from the vertical at distance x from its left endpoint, at time t, of a string of length L, fastened at both endpoints, satisfies the PDE

utt + aut = c2uxx, where a is a positive constant, with initial conditions

u(x, 0) = f(x), ut(x, 0) = g(x).

1. Solve the equation by separation of variables. The solution could depend on where a is, compared to the eigenvalues of the corresponding Sturm Liouville problem. That is, a could be less than all the eigenvalues, or between two of the eigenvalues.
2. Let λ be the first eigenvalue of the corresponding Sturm-Liouville problem. There is a qualitative difference in the behavior of the string if a > 2 λ and the behavior for 0 < a < λ. Describe it.

3. What can be said about limt→∞ u(x, t)?