**Consider the following problem; it can be interpreted as modeling the temperature distribution along a rod of length 1 with temperature decreasing along every point of the rod at a rate of bx (x the distance from the left endpoint, b a constant) while a heat source increases at each point the temperature by a rate proportional to the temperature at that point. In addition, the endpoints of the rod are kept at a temperature of 0. The initial temperature distribution is f(x).**

1. **ut  = uxx + ((pi)2 /4)\*u – bx , 0 < x < 1, t>0**

**u(0,t) = 0 t>0**

**u(1,0) = 0 t>0**

**u(x,0) 0 < x < 1**

1. **Determine the steady state (equilibrium) solution. This will require solving a relatively simple ODE (linear, second order; use undetermined coefficients). Your final answer should be**

**UE(x) = (4b/(pi)2)\*(x-sin((Pi/2)\*x)**

**b)Let v(x, t) = u(x, t) − (x). Show that v satisfies the following problem:**

**(2) vt  = vxx + ((pi)2 /4)\*v , 0 < x < 1, t>0**

**v(0,t) = 0 t>0**

**v(1,0) = 0 t>0**

**v(x,0) = g(x) 0 < x < 1**

**Determine how g(x) is related to f(x).**

**c) Let w(x, t) = e –((Pi)2\*t/4)\*v(t) show that w satisfies**

**(3) wt  = wxx , 0 < x < 1, t>0**

**w(0,t) = 0 t>0**

**w(1,t) = 0 t>0**

**w(x,0) = h(x) 0 < x < 1**

**Determine how h(x) is related to f(x).**

**(d) Solve equation (2) by separation of variables.**

**(e) Write out the solution of equation (1) for f(x) = uo , uo a constant. Verify that**

**Lim u(x,t) = uE(x,t)**

**t🡪infinity**

**Solve (2) (part b) to get v(x,t). Clearly state the formula for the coefficients and explain how this formula is obtained.**

 wt

w(0,t)  w(1, t) w(x,0)

= wxx, 0<x<1, t>0, = 0, t>0, =0, t>0, = h(x), 0<x<1.

Determine how h(x) is related to f(x).

(f )  
3. Consider the following eigenvalue problem:

Determine the solution u(x, t) of problem (1) and show that limt→∞ u(x, t) = uE (x, t) for all x, 0 < x < 1.