Consider the following problem

ut = 4uxx  0<x<Pi, t>0

u(0,t)=a(t), u(Pi,t)=b(t) t>0

u(x,0)=f(x) 0<x<Pi

1. Show that the solution (which exists and is unique for reasonably nice functions f,a,b) u(x,t) is of the form

U(x,t) = v(x,t)+(1-x/Pi) a(t)+x/Pi b(t)

where v solves a heat equation of the form vt = 4vxx + q(x, t) with homogeneous boundary conditions:

v(0,t) = v(π,t) = 0 for t > 0. Determine q(x,t).

* (b)  Assume a(t) ≡ a0,b(t) ≡ b0 are constant. Determine the steady state solution uE. How does this  solution depend on the initial value f(x)?
* (c)  Show that for large t one has u(x, t) ≈ uE (t) + C e−4t sin x, for some constant C . Determine C .