9. Why must any reversible heat engine eject waste heat rather than converting all the heat it absorbs into work?

Show, from the second law of thermodynamics, that all reversible heat engines have the same efficiency, and that no engine can exceed this efficiency. Quote without proof the efficiency of a reversible heat engine that absorbs heat from a reservoir at temperature T_1 and ejects it to a reservoir at temperature T_2.

A building is maintained at temperature T_b by pumping heat from a pond at temperature T_2. The heat exchanger in the pond is perfectly efficient, but in the one that heats the air in the building there is a temperature difference ΔT between the temperatures of the output from the heat pump and the air. ΔT is related to the power P of the fan that blows air over the heat exchanger by

$$\Delta T = T_0 \mathrm{e}^{-P/P_0},$$

where T_0 and P_0 are constants. Heat leaks from the building at a rate H. Draw a schematic diagram that shows the flow of energy into and out of the building.

Show that the total electrical power required for the system is

$$\eta(H - P) + P \quad \text{where} \quad \eta = 1 - \frac{T_2}{T_\mathrm{b} + T_0 \mathrm{e}^{-P/P_0}}\,.$$

Show that the system is cheapest to run when P satisfies

$$\frac{H - P}{P_0} - 1 = \frac{T_\mathrm{b}}{T_0} \mathrm{e}^{P/P_0}.$$

For the case in which $T_\mathrm{b} = 290\,\mathrm{K}$, $T_0 = 3\,\mathrm{K}$, $H = 1\,\mathrm{MW}$ and $P_0 = 1\,\mathrm{kW}$, determine whether the value of P that satisfies this equation is smaller than $1\,\mathrm{kW}$.