[1] A random sample of the birth weights of 186 babies has a mean of 3103g and a standard deviation of 696g (based on data from ``Cognitive Outcomes of Preschool Children with Prenatal Cocaine Exposure,'' by Singer et al., *Journal of the American Medical Association*, Vol. 291, No. 20. These babies were born to mothers who did not use cocaine during their pregnancies. Further, a random sample of the birth weights of 190 babies born to mothers who used cocaine during their pregnancies has a mean of 2700g and a standard deviation of 645g.

1. Set up the null and alternative hypotheses to test the claim that both samples are from the populations having the same standard deviation.

We want to test whether the populations have the same standard deviation or not. This is a test for two population standard deviation/variance so I have used the F Test.

Null Hypothesis (Ho): σ12 = σ22

Alternative Hypothesis (Ha): σ12 ≠ σ22

Let σ12 be the population standard deviation of mothers who did not use cocaine during their pregnancies and σ22 be the population standard deviation of mothers who used cocaine during their pregnancies.

1. Perform the hypothesis test you set up in a) with the significance level  = 0.05.

|  |  |
| --- | --- |
| **F Test for Differences in Two Variances** |  |
|  |  |
| **Data** | |
| **Level of Significance** | **0.05** |
| **Larger-Variance Sample** |  |
| **Sample Size** | **186** |
| **Sample Variance** | **484416** |
| **Smaller-Variance Sample** |  |
| **Sample Size** | **190** |
| **Sample Variance** | **416025** |
|  |  |
| Intermediate Calculations | |
| ***F* Test Statistic** | **1.1644** |
| Population 1 Sample Degrees of Freedom | 185 |
| Population 2 Sample Degrees of Freedom | 189 |
|  |  |
| **Two-Tail Test** |  |
| **Upper Critical Value** | **1.3329** |
| ***p*-Value** | **0.2986** |
| **Do not reject the null hypothesis** |  |

Since the critical value is still larger than the test statistic, we fail to reject the null hypothesis. Hence, there is insufficient evidence that the population variances (standard deviations) are not equal.

1. Using the results of b), does cocaine use appear to affect the birth weight of a baby? Substantiate you conclusion.

Since the p-value of the test is bigger than 0.05 level of significance so we fail to reject the null hypothesis. The data does not provide enough evidence to reject the claim that ~~both samples are~~ f~~rom~~ the populations ~~having~~ have the same standard deviation. Thus, we conclude that cocaine use does not appear to affect the birth weight of a baby.

You haven’t proved anything about this claim. From b), you showed that the population variances are not different. Now you need to prove that the mean weights are different using the conditions from (b).

[2] Simple random samples of high-interest (5.36%) mortgages and low-interest (3.77%) mortgage were obtained. For the 40 high-interest mortgages, the borrowers had a mean FICO credit score of 594.8 and standard deviation of 12.2. For the 40 low-interest mortgages, the borrowers had a mean FICO credit score of 785.2 and standard deviation of 16.3.

1. Use a 0.01 significance level to test the claim that the mean FICO score of borrowers with high-interest mortgage is lower than the mean FICO score of borrowers with low-interest mortgage.

We shall apply the Z-test because the standard deviations are known for both, the samples are large enough so we can assume the normality by applying the Central Limit Theorem.

Null Hypothesis (Ho): µ1 ≥ µ2

Alternative Hypothesis (Ha): µ 1 < µ2

Let µ1 be the population mean FICO score of borrowers with high-interest mortgage and µ2 be the population mean FICO score of borrowers with low-interest mortgage

1. Does the FICO credit rating score appear to affect mortgage payments? If so, how?

|  |  |
| --- | --- |
| **Z Test for Differences in Two Means** |  |
|  |  |
| **Data** | |
| **Hypothesized Difference** | **0** |
| **Level of Significance** | **0.01** |
| **Population 1 Sample** | |
| **Sample Size** | **40** |
| **Sample Mean** | **594.8** |
| **Population Standard Deviation** | **12.2** |
| **Population 2 Sample** | |
| **Sample Size** | **40** |
| **Sample Mean** | **785.2** |
| **Population Standard Deviation** | **16.3** |
|  |  |
| Intermediate Calculations | |
| Difference in Sample Means | -190.4 |
| Standard Error of the Difference in Means | 3.2192 |
| *Z* Test Statistic | -59.1451 |
|  |  |
| **Lower-Tail Test** | |
| **Lower Critical Value** | **-2.3263** |
| ***p*-Value** | **0.0000** |
| **Reject the null hypothesis** | |

How could you know that Z-test is applicable? Do you know that the population standard deviations are known? In order to apply a Z-test, in general, the population standard deviation is known.

Since the p-value of the test is smaller than 0.01 level of significance so we will reject the null hypothesis. The data provides enough evidence to support the claim that mean FICO score of borrowers with high-interest mortgage is lower than the mean FICO score of borrowers with low-interest mortgage. Based on the above analysis, the higher FICO scores are associated with lower interest rates which means that the interest portion would be less in the total monthly mortgage payment and the opposite can also be expected. Thus, the FICO credit rating scores appear to affect mortgage payments.

\*Note: I didn’t know how to word this portion. I did not want to say the mortgage payment would lower because that could be affected by the size of the loan also and not only the interest rate.

What would you think about the mortgage rate you would be given if you had a low FICO score? Meanwhile I have a higher FICO score than yours, and got the same amount of loan. Do you think that you and I should pay the same monthly payment?

[3] A random sample of the birth weights of 186 babies has a mean of 3103g and a standard deviation of 696g (based on data from “Cognitive Outcomes of Preschool Children with Prenatal Cocaine Exposure,” by Singer et al., *Journal of the American Medical Association*, Vol. 291, No. 20). These babies were born to mothers who did not use cocaine during their pregnancies. Further, a random sample of the birth weights of 190 babies born to mothers who used cocaine during their pregnancies has a mean of 2700g and a standard deviation of 645g.

1. The birth weights of babies are known normally distributed. Use a 0.05 significance level to test the claim that both samples are from populations having the same standard deviation.

We shall apply the F test because this is two sample test for standard deviation or variance and the weights are known normally distributed.

Null Hypothesis (Ho): σ12 = σ22

Alternative Hypothesis (Ha): σ12 > σ22

This is an upper-tail test.

Let σ12 be the population variance for the population of birth weights of babies from mothers who did not use cocaine during their pregnancies and σ22 be the population variance for the population of birth weights of babies from mothers who used cocaine during their pregnancies.

|  |  |
| --- | --- |
| **F Test for Differences in Two Variances** |  |
|  |  |
| **Data** | |
| **Level of Significance** | **0.05** |
| **Larger-Variance Sample** |  |
| **Sample Size** | **186** |
| **Sample Variance** | **484416** |
| **Smaller-Variance Sample** |  |
| **Sample Size** | **190** |
| **Sample Variance** | **416025** |
|  |  |
| Intermediate Calculations | |
| ***F* Test Statistic** | **1.1644** |
| Population 1 Sample Degrees of Freedom | 185 |
| Population 2 Sample Degrees of Freedom | 189 |
|  |  |
| **Two-Tail Test** |  |
| **Upper Critical Value** | **1.3329** |
| ***p*-Value** | **0.2986** |
| **Do not reject the null hypothesis** |  |

Since the p-value of the test is bigger than 0.05 level of significance so we fail to reject the null hypothesis. The data does not provide enough evidence to reject the claim that both samples are from populations having the same standard deviation.

1. Using your finding in (a), construct a 90% confidence interval estimate of the difference between the mean birth weight of a baby born to mothers who did not use cocaine and that of a baby born to mothers who used cocaine during their pregnancies.

Based on our findings in a), the data does not provide enough evidence to reject the claim that both samples are from populations having the same standard deviation so we have applied the pooled-variance t test.

|  |  |
| --- | --- |
| **Pooled-Variance *t* Test for the Difference Between Two Means** | |
| (assumes equal population variances) |  |
| **Data** | |
| **Hypothesized Difference** | **0** |
| **Level of Significance** | **0.01** |
| **Population 1 Sample** |  |
| **Sample Size** | **40** |
| **Sample Mean** | **594.8** |
| **Sample Standard Deviation** | **12.2** |
| **Population 2 Sample** |  |
| **Sample Size** | **40** |
| **Sample Mean** | **785.2** |
| **Sample Standard Deviation** | **16.3** |
|  |  |
| Intermediate Calculations | |
| Population 1 Sample Degrees of Freedom | 39 |
| Population 2 Sample Degrees of Freedom | 39 |
| Total Degrees of Freedom | 78 |
| Pooled Variance | 207.2650 |
| Standard Error | 3.2192 |
| Difference in Sample Means | -190.4000 |
| ***t* Test Statistic** | **-59.1451** |
|  |  |
| **Lower-Tail Test** |  |
| **Lower Critical Value** | **-2.3751** |
| ***p*-Value** | **0.0000** |
| **Reject the null hypothesis** |  |

This is not in question. This type of test should go to [1] (c).

|  |  |
| --- | --- |
| **Confidence Interval Estimate** | |
| **for the Difference Between Two Means** | |
|  |  |
| **Data** | |
| **Confidence Level** | **90%** |
|  |  |
| Intermediate Calculations | |
| Degrees of Freedom | 374 |
| *t* Value | 1.6489 |
| Interval Half Width | 114.0778 |
|  |  |
| **Confidence Interval** | |
| **Interval Lower Limit** | **288.9222** |
| **Interval Upper Limit** | **517.0778** |

1. Using your finding in (b), does cocaine use appear to affect the birth weight of a baby? Substantiate you conclusion.

We can see that 0 does not lie inside the 90% confidence interval. Based on the findings we can conclude that we are 90% confident that cocaine use appears to affect the birth weight of a baby.

[4] A large discount chain compares the performance if its credit managers in Ohio and Illinois by comparing the mean dollars amounts owed by customers with delinquent charge accounts in these two states. Here a small mean dollar amount owed is desirable because it indicates that bad credit risks are not being extended large amounts of credit. Two independent, random samples of delinquent accounts are selected from the populations of delinquent accounts in Ohio and Illinois, respectively. The first sample, which consists of 15 randomly selected delinquent accounts in Ohio, givens a mean dollar amount of $524 with a standard deviation of $38. The second sample, which consists of 20 randomly selected delinquent accounts in Illinois, gives a mean dollar amount of $473 with a standard deviation of $22.

1. Assuming that the normality assumption, test to determine if the population variances are equal with  =0.05.

Since this is a two sample test for variance and normality assumption is valid, we can use the F test for differences in two variances.

Null Hypothesis (Ho): σ12 = σ22

Alternative Hypothesis (Ha): σ12 ≠ σ22

Let σ12 be the population standard deviation of Ohio and σ22 be the population standard deviation of Illinois.

|  |  |
| --- | --- |
| **F Test for Differences in Two Variances** |  |
|  |  |
| **Data** | |
| **Level of Significance** | **0.05** |
| **Larger-Variance Sample** |  |
| **Sample Size** | **15** |
| **Sample Variance** | **1444** |
| **Smaller-Variance Sample** |  |
| **Sample Size** | **20** |
| **Sample Variance** | **484** |
|  |  |
| Intermediate Calculations | |
| ***F* Test Statistic** | **2.9835** |
| Population 1 Sample Degrees of Freedom | 14 |
| Population 2 Sample Degrees of Freedom | 19 |
|  |  |
| **Two-Tail Test** |  |
| **Upper Critical Value** | **2.6469** |
| ***p*-Value** | **0.0283** |
| **Reject the null hypothesis** |  |

Since the p-value of the test is smaller than 0.05 level of significance so we will reject the null hypothesis. The data provides enough evidence to support that both samples are from populations having different variances.

1. Test with  =0.05 whether there is a difference between the population mean dollar amounts owed by consumers with delinquent charge accounts in Ohio and Illinois.

Based on our findings in a), the population variances are different so we shall apply a separate-variances t test.

Null Hypothesis (Ho): µ1 = µ2

Alternative Hypothesis (Ha): µ 1 ≠ µ2

Let µ1 be population mean dollar amounts owed by consumers with delinquent charge accounts in Ohio and let µ2 be the population mean dollar amounts owed by consumers with delinquent charge accounts in Illinois

|  |  |
| --- | --- |
| **Separate-Variances *t* Test for the Difference Between Two Means** | |
| (assumes unequal population variances) |  |
| **Data** | |
| **Hypothesized Difference** | **0** |
| **Level of Significance** | **0.05** |
| **Population 1 Sample** |  |
| **Sample Size** | **15** |
| **Sample Mean** | **524** |
| **Sample Standard Deviation** | **38.0000** |
| **Population 2 Sample** |  |
| **Sample Size** | **20** |
| **Sample Mean** | **473** |
| **Sample Standard Deviation** | **22.0000** |
|  |  |
| Intermediate Calculations | |
| Numerator of Degrees of Freedom | 14512.2178 |
| Denominator of Degrees of Freedom | 692.7711 |
| Total Degrees of Freedom | 20.9481 |
| Degrees of Freedom | 20 |
| Standard Error | 10.9757 |
| Difference in Sample Means | 51.0000 |
| **Separate-Variance *t* Test Statistic** | **4.6466** |
|  |  |
| **Two-Tail Test** |  |
| **Lower Critical Value** | **-2.0860** |
| **Upper Critical Value** | **2.0860** |
| ***p*-Value** | **0.0002** |
| **Reject the null hypothesis** |  |

Since the p-value of the test is smaller than 0.05 level of significance so we will reject the null hypothesis. The data provides enough evidence to support that there is a difference between the population mean dollar amounts owed by consumers with delinquent charge accounts in Ohio and Illinois.

1. Assuming that the normality assumption and the condition you checked in a) hold, calculate a 95 percent confidence interval for the difference between the mean dollar amounts owed in Ohio and Illinois. Based on this interval, do you think that these mean dollar amounts differ in a practically important way?

|  |  |
| --- | --- |
| **Confidence Interval Estimate** | |
| **for the Difference Between Two Means** | |
|  |  |
| **Data** | |
| **Confidence Level** | **95%** |
|  |  |
| Intermediate Calculations | |
| Degrees of Freedom | 33 |
| *t* Value | 2.0345 |
| Interval Half Width | 20.7463 |
|  |  |
| **Confidence Interval** | |
| **Interval Lower Limit** | **30.2537** |
| **Interval Upper Limit** | **71.7463** |

We can see that the lower limit of the 95% confidence interval is bigger than 0, it means we can conclude with 95% confidence that the mean dollar amounts owed in Ohio is higher as compared to Illinois. Since 0 does not include in 95% confidence interval so we can conclude that these mean dollar amounts differ in a practically important way.

I don’t believe that PHStat provides a confidence interval estimate under the condition (b) where you state that we assume unequal population variances.

PHStat can only calculate such an interval under the equal variance assumption like you did in [3] (b).

Further, PHStat is not almighty, and if you wish, you can try SSBEC for the last problem.