http://ramanujan.math.trinity.edu/wtrench/texts/TRENCH_LAGRANGE_MULTIPLIERS.PDF

the extrema subject to two constraints is on page 11 in the link above

5 Extrema subject to two constraints

Here is Theorem 1 with $m = 2$.

Theorem 3 *Suppose that $n > 2$. If $\mathbf{X}_0$ is a local extreme point of f subject to $g_1(\mathbf{X}) = g_2(\mathbf{X}) = 0$ and*

$$\left| \begin{array}{cc} \dfrac{\partial g_1(\mathbf{X}_0)}{\partial x_r} & \dfrac{\partial g_1(\mathbf{X}_0)}{\partial x_s} \\ \dfrac{\partial g_2(\mathbf{X}_0)}{\partial x_r} & \dfrac{\partial g_2(\mathbf{X}_0)}{\partial x_s} \end{array} \right| \neq 0 \qquad (19)$$

for some r and s in $\{1, 2, \dots, n\}$, then there are constants λ and μ such that

$$\frac{\partial f(\mathbf{X}_0)}{\partial x_i} - \lambda \frac{\partial g_1(\mathbf{X}_0)}{\partial x_i} - \mu \frac{\partial g_2(\mathbf{X}_0)}{\partial x_i} = 0, \qquad (20)$$

$1 \leq i \leq n.$

Proof For notational convenience, let $r = 1$ and $s = 2$. Denote

$$\mathbf{U} = (x_3, x_4, \dots x_n) \text{ and } \mathbf{U}_0 = (x_{30}, x_{30}, \dots x_{n0}).$$

Since

$$\left| \begin{array}{cc} \dfrac{\partial g_1(\mathbf{X}_0)}{\partial x_1} & \dfrac{\partial g_1(\mathbf{X}_0)}{\partial x_2} \\ \dfrac{\partial g_2(\mathbf{X}_0)}{\partial x_1} & \dfrac{\partial g_2(\mathbf{X}_0)}{\partial x_2} \end{array} \right| \neq 0, \qquad (21)$$

the Implicit Function Theorem (Theorem 6.4.1, p. 420) implies that there are unique continuously differentiable functions

$$h_1 = h_1(x_3, x_4, \dots, x_n) \text{ and } h_2 = h_1(x_3, x_4, \dots, x_n),$$

defined on a neighborhood $N \subset \mathbb{R}^{n-2}$ of $\mathbf{U}_0$, such that $(h_1(\mathbf{U}), h_2(\mathbf{U}), \mathbf{U}) \in D$ for all $\mathbf{U} \in N$, $h_1(\mathbf{U}_0) = x_{10}$, $h_2(\mathbf{U}_0) = x_{20}$, and

Can you point where all the mathematical notation underlined with violet above is in the implicit function since they use that proof. That is show where h_1 h_2, **U**, x_{10}, x_{20}, N and $\mathbb{R}^{n-2}$ is in the general formulation of the implicit function theorem And why have they defined $h_2=h_1$?:

Theorem 6.4.1 (The Implicit Function Theorem) *Suppose that $\mathbf{F} : \mathbb{R}^{n+m} \to \mathbb{R}^m$ is continuously differentiable on an open set S of $\mathbb{R}^{n+m}$ containing $(\mathbf{X}_0, \mathbf{U}_0)$. Let $\mathbf{F}(\mathbf{X}_0, \mathbf{U}_0) = \mathbf{0}$, and suppose that $\mathbf{F}_\mathbf{U}(\mathbf{X}_0, \mathbf{U}_0)$ is nonsingular. Then there is a neighborhood M of $(\mathbf{X}_0, \mathbf{U}_0)$, contained in S, on which $\mathbf{F}_\mathbf{U}(\mathbf{X}, \mathbf{U})$ is nonsingular and a neighborhood N of $\mathbf{X}_0$ in $\mathbb{R}^n$ on which a unique continuously differentiable transformation $\mathbf{G} : \mathbb{R}^n \to \mathbb{R}^m$ is defined, such that $\mathbf{G}(\mathbf{X}_0) = \mathbf{U}_0$ and*

$$(\mathbf{X}, \mathbf{G}(\mathbf{X})) \in M \quad \text{and} \quad \mathbf{F}(\mathbf{X}, \mathbf{G}(\mathbf{X})) = 0 \quad \text{if} \quad \mathbf{X} \in N. \qquad (6.4.6)$$

Moreover,

$$\mathbf{G}'(\mathbf{X}) = -[\mathbf{F}_\mathbf{U}(\mathbf{X}, \mathbf{G}(\mathbf{X}))]^{-1} \mathbf{F}_\mathbf{X}(\mathbf{X}, \mathbf{G}(\mathbf{X})), \quad \mathbf{X} \in N. \qquad (6.4.7)$$

Search for the theorem in this link if you want to see more from the implicit function theorem. It is on page 420

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