MODULE TITLE : CONTROL SYSTEMS AND AUTOMATION

TOPIC TITLE : MODELLING OF PROCESSES

LESSON 2 : LAPLACE TRANSFORMS AND SIMULATION

CSA - 3 - 2

Published by Teesside University Open Learning (Engineering)
School of Science & Engineering
Teesside University
Tees Valley, UK
TS1 3BA
+44 (0)1642 342740

INTRODUCTION

In our studies so far we have often ended up with differential equations which represent the modelling and control of processes. These equations can be very complex and prove difficult to solve by the normal integration methods studied so far (bear in mind that, in this module, we have only dealt with simple examples of processes).

There is a mathematical procedure in which we convert, or transform, a problem which is difficult to solve in its present form into another form in which it is easier to solve. This procedure is known as **transformation**. It is widely used in Mathematics and Control Engineering. Once the problem has been solved, the solution is converted back into the original form. This is known as **inverse transformation**.

There are many kinds of transformation, each especially adapted to the type of problem to be solved. For solving differential equations, a common transformation used is known as the **Laplace transformation**. This transforms an awkward differential equation into an easily solved algebraic equation. The solution provides both the transient and steady state responses of the system described by the differential equation. Further, any initial conditions of the system are easily incorporated into the algebraic equation. Laplace transformation can be done mathematically or, as is more common, by reference to tables of transforms.

With the aid of Laplace transforms, it is possible to produce a simulation of the Process and possible control systems using computer software (ranging from simple Microsoft Excel spreadsheets to highly developed specialised software) to run the process and test the control system and settings to see the effect of changes before the plant is actually built and run. We will look at a simple example of a simulation of a process and its control system to show the possible benefits simulation brings to the design engineer.

YOUR AIMS

On completion of this lesson, you should be able to:

- explain how the Laplace transformation can be used to transform a differential equation into one which is algebraic
- use a table of transforms to solve a simple differential equation
- explore a simple simulation software program to show the effect of changes to the process and to its control system parameters on the process.

STUDY ADVICE

We will be using Partial Fractions within this lesson. You may wish to revise them from Analytical Methods for Engineers (AME - 3 - 13).

INTRODUCTION

In the days before calculators were in everyday use engineers and mathematicians used to carry out complex calculations using log tables (some of you may still use them). Most people did not fully understand the way in which they were produced but knew they produced correct results (if used correctly). The numbers you had were looked up in the log table and transformed into different numbers which you then treated according to set rules to give you an answer. This answer was then looked up in the antilog tables to give you the final answer you required. This is one example of a mathematical transformation process.

Thinking now of the simple calculator. You enter a number into it, press the relevant operator (+,–, ×, ÷, etc.), enter the next number and press =. The result is shown on the display. Do you know how it does this? Does this stop you from using it to aid your calculations?

In very simple terms, the numbers you enter are transformed into a digital electrical code, pressing the operator selects the circuit the numbers are to be processed by, the = sign transforms the signal leaving the circuit back into the result to be shown on the display. All this before you have time to think! This is another example of a transformation process.

A further example of a mathematical transformation is the Laplace transform that if used correctly can help us solve some mathematical problems much more quickly and easily than doing it 'long hand'.

In the last lesson, we developed differential equations as part of the mathematical modelling of a process. We also met differential equations in control systems. The Laplace Transform is a method used to convert these differential equations into simpler algebraic equations in a different domain which can be manipulated and solved easily and then the answers transformed back into the original domain using inverse Laplace transforms.

Let's look at a simple example to show its usefulness. Consider a simple thermocouple whose detecting element is placed directly into a fluid (i.e. in direct contact with the fluid) and left to reach equilibrium with the fluid. Initially at a time t = –0 (i.e. just before we start the timing), the sensor temperature (T_s) and the fluid temperature (T_f) will be the same i.e. $T_{s,-0} = T_{f,-0}$.

If the fluid temperature is suddenly raised at t = 0, the sensor is no longer in a steady state and we can represent the sensor's dynamic behaviour by the heat balance equation:

Rate of heat inflow – rate of heat outflow
= rate of change of sensor heat content

Looking at this equation:

- Assuming $T_f > T_s$ then the heat flow from the sensor can be regarded as equal to 0.

- The rate of heat inflow (Q) will be proportional to the temperature difference between fluid and sensor ($T_f - T_s$) and is given by;

$$Q = UA\left(T_f - T_s\right)$$

where U = overall heat transfer coefficient for the system $\left(\text{W m}^{-2}\ {}^\circ\text{C}^{-1}\right)$
A = heat transfer area $\left(\text{m}^2\right)$.

This will vary with time as the sensor heats up.

If we define ΔT_s as $(T_s - T_{s,-0})$ and ΔT_f as $(T_f - T_{f,-0})$, then at any point in time;

$$Q = UA\left(\Delta T_f - \Delta T_s\right)$$

- The increase in heat content of the sensor is given by $MC(T_s - T_{s,-0})$

where M = mass of sensor
C = specific heat capacity of the sensor material
$\left(T_s - T_{s,-0}\right)$ = the temperature increase of the sensor over time t.

Thus,

$$Q\text{, the rate of change of sensor heat content} = MC\frac{\mathrm{d}\left(T_s - T_{s,-0}\right)}{\mathrm{d}t}$$

$$= MC\frac{\mathrm{d}\Delta T_s}{\mathrm{d}t}$$

and equating we get:

$$UA\left(\Delta T_f - \Delta T_s\right) = MC\frac{\mathrm{d}\Delta T_s}{\mathrm{d}t}$$

Which on rearrangement gives us:

$$\frac{MC}{UA}\frac{\mathrm{d}\Delta T_s}{\mathrm{d}t} + \Delta T_s = \Delta T_f$$

This is a linear differential equation of the first order; the derivative term is the first derivative $\frac{\mathrm{d}\Delta T_\mathrm{s}}{\mathrm{d}t}$. If you work out the value of the constant $\left(\frac{MC}{UA}\right)$ in terms of units you will find it has the units of time and can be replaced by a single value of τ seconds so the equation becomes;

$$\tau \frac{\mathrm{d}\Delta T_\mathrm{s}}{\mathrm{d}t} + \Delta T_\mathrm{s} = \Delta T_\mathrm{f}$$

If we look at this equation in more detail we could say that the output of the thermocouple (the change in ΔT_s) is being forced by the input change ΔT_f both of which are dependent upon time. This means that the equation cannot be solved by conventional integration methods and we have to use a new mathematical technique – the Laplace transform.

The equation can be transformed into a simple algebraic equation using the Laplace transform. This transforms the equation into what is known as the *s* domain from the time (*t*) domain.

The **Laplace transform** $F(s)$ of a time function f(t) is defined to be:

$$F(s) = L\{f(t)\} = \int_0^\infty f(t)\mathrm{e}^{-st}\mathrm{d}t \quad \text{........................} \quad (1)$$

where L indicates the process of Laplace transformation of the time function $f(t)$, i.e. the expression $L\{f(t)\}$ is to be read as 'the Laplace transform of $f(t)$'.

s is the Laplace variable of the new domain. That is, whereas the differential equations of physical systems involve t, the Laplace transformed algebraic equations will involve s. It is a complex variable of the form:

$$s = \sigma + j\omega \qquad \left(\text{where } j = \sqrt{-1}\right)$$

For the purposes of this module, we need only a limited acquaintance with the theory of this transform and will thus only consider the outcome. TABLE 1 gives the Laplace transforms of some common standard functions $f(t)$.

One important property of Laplace transforms is that they are linear, i.e.

$$L\{f_1(t) + f_2(t)\} = F_1(s) + F_2(s)$$

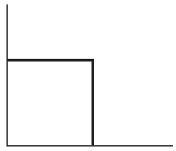
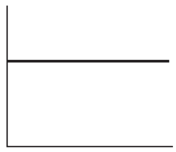
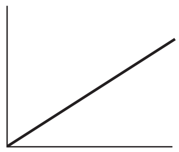
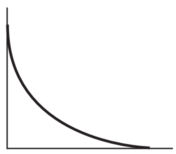
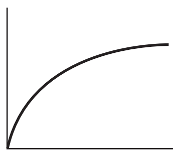
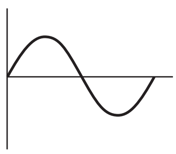
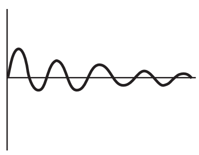
	Symbol	*Graph*	*Laplace Transform F(s)*
Function of time f(t)	$Kf(t)$		$KF(s)$
1st Derivative	$\frac{df(t)}{dt}$		$s\,F(s)-f(-0)$
Unit impulse	$\delta(t)$		1
Unit step	$u(t)$		$\frac{1}{s}$
Ramp	t		$\frac{1}{s^2}$
Exponential decay	e^{-at}		$\frac{1}{s+a}$
Exponential growth	$1-e^{-at}$		$\frac{a}{s\,(s+a)}$
Sine wave	$Sin\ \omega t$		$\frac{\omega}{(s^2+\omega^2)}$
Exponentially damped sine wave	$e^{-at}\sin \omega t$		$\frac{\omega}{(s+a)^2+\omega^2}$

TABLE 1

Let's go back to the earlier equation we developed for the thermocouple in the fluid to see how the Laplace transform can be used, i.e.

$$\tau \frac{\mathrm{d}\Delta T_{\mathrm{s}}}{\mathrm{d}t} + \Delta T_{\mathrm{s}} = \Delta T_{\mathrm{f}}$$

We can carry out a Laplace Transform on it, term by term, to give

$$\tau\left[s\Delta\overline{T}_{\mathrm{s}}(s) - \Delta T_{-0}\right] + \Delta\overline{T}_{\mathrm{s}}(s) = \Delta\overline{T}_{\mathrm{f}}(s)$$

where $\Delta\overline{T}$ is the Laplace transform of ΔT

ΔT_{-0} = the temperature difference at initial conditions immediately prior to $t = 0$ (if you remember we set this to be 0).

So $$\tau\left[s\Delta\overline{T}_{\mathrm{s}}(s)\right] + \Delta\overline{T}_{\mathrm{s}}(s) = \Delta\overline{T}_{\mathrm{f}}(s)$$

or $$\Delta\overline{T}_{\mathrm{s}}(s)\left[\tau s + 1\right] = \Delta\overline{T}_{\mathrm{f}}(s)$$

If we consider $\Delta\overline{T}_{\mathrm{s}}(s)$ as the output and $\Delta\overline{T}_{\mathrm{f}}(s)$ as the input we can determine the **transfer function(output/input)** or **gain** for this system in the s domain as

$$G(s) = \frac{\Delta\overline{T}_{\mathrm{s}}(s)}{\Delta\overline{T}_{\mathrm{f}}(s)} = \frac{1}{\tau s + 1}$$

and can represent this on a block diagram (compare this with our earlier work in Topic 1 and 2 on control systems diagrams).

$$\Delta\overline{T}_{\mathrm{f}}(s) \longrightarrow \boxed{\frac{1}{\tau s + 1}} \longrightarrow \Delta\overline{T}_{\mathrm{s}}(s)$$

Now
$$\frac{\Delta \overline{T}_s(s)}{\Delta \overline{T}_f(s)} = \frac{1}{\tau s + 1}$$

However, we gave as our initial conditions, at $t = 0$, that the temperature of the fluid was suddenly changed (i.e. there was a step change in temperature) by an amount ΔT_f. According to our table of transforms, for a step change of ΔT_f, the transform $\Delta \overline{T}_f(s)$ is $\frac{\Delta T_f}{s}$.

So
$$\Delta \overline{T}_s(s) = \frac{\Delta \overline{T}_f(s)}{\tau s + 1} = \frac{\frac{\Delta T_f}{s}}{\tau s + 1} = \frac{\Delta T_f}{s(\tau s + 1)}$$

$$= \frac{\Delta T_f}{\tau s\left(s + \frac{1}{\tau}\right)}$$

If we look at our table of transforms given earlier we might be able to identify that the transform $\frac{1}{\tau s\left(s + \frac{1}{\tau}\right)}$ is of the same general form as $\frac{a}{s(s + a)}$ $\left(\text{where } a = \frac{1}{\tau}\right)$ which is the Laplace transform of the function $1 - e^{-at}$.

So what about going in the opposite direction and going back into the time domain from the s domain? Can we do it? The answer is yes and it is known as inverse transformation.

$\frac{1}{\tau s\left(s + \frac{1}{\tau}\right)}$ when transformed becomes $1 - e^{-at} = 1 - e^{\frac{-t}{\tau}}$ since $a = \frac{1}{\tau}$.

Hence
$$\Delta T_s = \Delta T_f\left(1 - e^{-\frac{t}{\tau}}\right)$$

This is the result you would predict for a step change, i.e. the sensor temperature would exponentially grow to the same temperature as the fluid, the rate of growth being determined by the time constant of the sensor, giving the method credibility.

Can you repeat the transform (not the inverse transform), but this time assuming that the temperature change of the fluid is a ramp input change given by $\Delta T_f = 0.5t$ °C (t = time in seconds)? Our answer follows.

The method is the same up until the transfer function stage, i.e

$$\frac{\Delta\overline{T}_{\mathrm{s}}(s)}{\Delta\overline{T}_{\mathrm{f}}(s)} = \frac{1}{\tau s + 1}$$

This time $\Delta T_{\mathrm{f}} = 0.5t$ and the transfer function for this ramped input is $\frac{0.5}{s^2}$.

Thus

$$\Delta\overline{T}_{\mathrm{s}}(s) = \frac{\Delta\overline{T}_{\mathrm{f}}(s)}{\tau s + 1} = \frac{\frac{0.5}{s^2}}{\tau s + 1} = \frac{0.5}{s^2(\tau s + 1)}$$

$$= \frac{0.5}{\tau s^2\left(s + \frac{1}{\tau}\right)}$$

$$= \frac{0.5}{s^2} \times \frac{\frac{1}{\tau}}{\left(s + \frac{1}{\tau}\right)}$$

This gives us

$$\Delta\overline{T}_{\mathrm{s}}(s) = \frac{0.5a}{s^2(s + a)}$$

where $a = \frac{1}{\tau}$ and the constant multiplier is 0.5.

The right-hand side of this equation is not in our list of transforms in TABLE 1, so how do we inverse transform back into the time domain? One method is to use partial fractions to separate the transform into two (or more) simpler transforms which *are* in the table.

If you look back at lesson AME - 3 - 13 you will find that for this type of fraction we can separate it into two partial fractions according to the standard form

$$\frac{a}{s^2(s + a)} = \frac{(As + B)}{s^2} + \frac{C}{(s + a)}$$

where A, B and C are constants.

Multiplying both sides by $s^2(s + a)$ we get

$$a = (As + B)(s + a) + Cs^2$$

Removing brackets

$$a = (As + B)(s + a) + Cs^2$$

$$a = As^2 + Bs + Aas + Ba + Cs^2$$

Equating the coefficients of same indices of s

s^0 $\quad a = Ba \qquad B = 1$

s^1 $\quad 0 = B + Aa$

Substituting for B from above $\qquad 0 = 1 + aA \qquad A = \dfrac{-1}{a}$

s^2 $\quad 0 = A + C \qquad A + C = 0 \qquad$ Substituting for A, $C = \dfrac{1}{a}$

Substituting these values back into the standard partial fraction equation, we get

$$\frac{a}{s^2(s + a)} = \frac{\left(-\frac{s}{a^2} + \frac{1}{a}\right)}{s^2} + \frac{\frac{1}{a^2}}{(s + a)}$$

$$= \frac{\left(-\frac{s}{a} + 1\right)}{s^2} + \frac{\frac{1}{a}}{(s + a)}$$

This can be simplified by separating off the $\dfrac{\left(-\dfrac{s}{a}+1\right)}{s^2}$ term into two fractions to

$$-\frac{\frac{s}{a}}{s^2}+\frac{1}{s^2}=-\frac{\frac{1}{a}}{s}+\frac{1}{s^2}$$

and this substituted back into the equation to give the final result

$$\frac{a}{s^2(s+a)}=-\frac{\frac{1}{a}}{s}+\frac{1}{s^2}+\frac{\frac{1}{a}}{(s+a)}$$

These three terms each have a transform in TABLE 1 so we can inverse transform back into the time domain for each term in turn:

$-\dfrac{\frac{1}{a}}{s}$ is inverse transformed into $-\dfrac{1}{\text{a}}$

$\dfrac{1}{s^2}$ is inverse transformed into t

$\dfrac{\frac{1}{a}}{(s+a)}$ is inverse transformed into $\dfrac{1}{a}\text{e}^{-at}$

Thus we can now inverse transform the equation we developed for the ramped input, i.e.

$$\Delta\overline{T_s}(s) = \frac{0.5a}{s^2(s + a)}$$

where $a = \dfrac{1}{\tau}$ and the constant multiplier is 0.5, into

$$\Delta T_s = 0.5\left[-\frac{1}{\frac{1}{\tau}} + t + \frac{1}{\frac{1}{\tau}}e^{-\frac{t}{\tau}}\right]$$

$$= 0.5\left[-\tau + t + \tau e^{-\frac{t}{\tau}}\right]$$

As before we can now calculate the answer to this equation to find the temperature of the sensor at a time t if we know the time constant of the sensor.

Try the following example for yourself.

A process can be represented by the first order equation

$$\tau \frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = Ku(t)$$

Determine the transfer function of the process.

If the input is a step change in $u(t)$, determine the value of $y(t)$ when $K = 6$, $t = 15$ s and $\tau = 10$ s.

Rearranging the equation

$$\frac{dy(t)}{dt} + \frac{1}{\tau}y(t) = \frac{Ku(t)}{\tau}$$

Using the Laplace transforms given in the earlier table

$$\left[sY(s) + y(-0)\right] + \frac{Y(s)}{\tau} = \frac{KU(s)}{\tau}$$

Assuming the initial condition was steady state ($y(-0) = 0$) we get

$$sY(s) + \frac{Y(s)}{\tau} = \frac{KU(s)}{\tau}$$

or $$Y(s)\left(s + \frac{1}{\tau}\right) = \frac{KU(s)}{\tau}$$

The transfer function in the s domain, $G(s)$, is input, $Y(s)$, over output, $U(s)$, so we can rearrange the above equation to give

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\frac{K}{\tau}}{\left(s + \frac{1}{\tau}\right)} = \frac{K}{\tau\left(s + \frac{1}{\tau}\right)}$$

$$= \frac{K}{\tau s + 1}$$

Having obtained the transfer function, you are now asked to determine the value of y(t).

To do this, we need to perform the inverse transform of $Y(s)$ to return to the time domain.

We therefore need to re-arrange the transfer function equation for when the input change is a step change i.e.

$$U(s) = \frac{1}{s}$$

$$Y(s) = \frac{\frac{K}{\tau}}{\left(s + \frac{1}{\tau}\right)} \times \frac{1}{s} = \frac{\frac{K}{\tau}}{s\left(s + \frac{1}{\tau}\right)}$$

This is now in the form of $\dfrac{a}{s(s + a)}$, where $a = \dfrac{1}{\tau}$.

The inverse transform of this is $1 - e^{-at}$ (from the table)

So

$$y(t) = K(1 - e^{-at}) = K\left(1 - e^{-\frac{t}{\tau}}\right)$$

Note, that you may have arrived at the same answer using a different route!

When $K = 6, \tau = 10$ s and $t = 15$ s

Then

$$y(15) = 6\left(1 - e^{-\frac{15}{10}}\right)$$

$$= 6 \times 0.777$$

$$= 4.662 \text{ units}$$

SIMULATION

In this and the previous topic we have been producing mathematical equations to represent processes and the action of control systems on them. We can, and do, solve these equations by various mathematical methods (the Laplace transform being one method we have used) and use the solutions to build up a 'picture' of how the process works and responds to changes. To attempt this manually would be a long and complex process which is prone to errors as you may well have found out! With the advent of spreadsheets and powerful computers, engineers began to look at solving the mathematical equations in a much faster way and using the results to produce real pictures; that is, creating **simulations** of the real process. They could then test out various situations that could occur on their design, whilst still at the design stage. This has a great effect on efficiency, costs and safety. Simulations of processing systems are now part of nearly all design processes, not just those on chemical plant and control systems.

The computer games industry adapted the same techniques, and developed more of their own, and has been one of the main drivers for the improved quality of simulated processes. Even simple computer card games produce useful benefits such as understanding how to play the game, how to improve your skill, the effect of different techniques or strategies, etc. (as well as providing entertainment!).

Think back 30 years (for those of us that can do this). The tennis game that was the rage then had a black TV screen with two cursor shaped bats that moved up and down the edge of the TV screen and a 'ball' that simply moved between them and bounced off the sides of the screen. The players moved the bats up and down using a joystick and lost a point if the ball went past the bat. Nowadays, you have full colour at your choice of venue (Wimbledon, etc.) with 3D graphics such that you can sometimes believe you are actually there, serving for the match – virtual reality. You can even control the speed and direction of your serve using wireless handsets.

You may have seen simulation being used in the design of kitchens where computer graphics turn your existing kitchen into your 'new' kitchen and you can 'virtually' walk around it to check if the colours are right (change them), the position of sockets and/or appliances is right (move them), verify the best height of wall units, etc. You may have to pay a little extra for the service but you should end up with the 'perfect' kitchen. A domestic example of the power of simulation.

Simulations are particularly useful when a real-life process:

- is dangerous
- takes too long
- is too quick to study
- is expensive to create.

We will look at a simple simulation example of a Process and its control system at a very basic level to show the principle, but bear in mind that much more sophisticated systems are available which can simulate whole chemical plants; maybe you have even been trained up for a job using such a simulator. Now let us examine the underlying theory.

The system we are going to consider is a process which comprises an electrical heater of heating capacity C_h connected via a thermal resistance R_o to an oven of heat capacity C_o. The oven loses heat to the environment, which is at temperature T_e, through the thermal resistance R_{ho} of its insulation. The temperature controller adjusts the power dissipated in the heating elements, M, by comparing the oven temperature, T_o, with the set-point temperature T_s. This can be represented by the simple electrical circuit shown opposite in FIGURE 1.

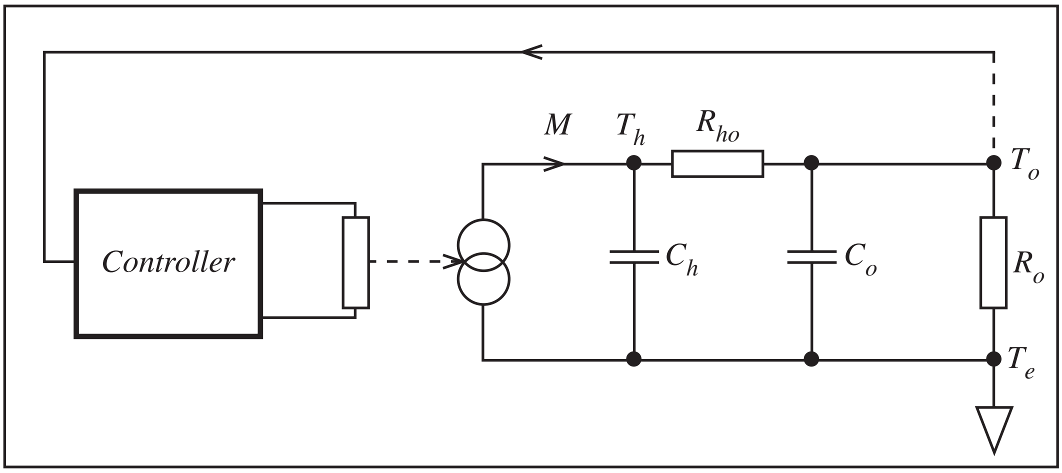


FIG. 1

This system can be modelled using the techniques described in the previous lesson and the mathematical equations solved, using excel spreadsheets or other software, quickly and easily for both on-off and PID (Proportional, Integral and Derivative) control to produce a mathematical simulation. The results can then be displayed graphically.

Let's consider **On-Off Control** for this oven. This is the simplest form of control, used by almost all domestic ovens which have a simple thermostat. When the oven is cooler than the set-point temperature the heater is turned on at maximum power, M, and once the oven is hotter than the set-point temperature the heater is switched off completely. The turn-on and turn-off temperatures are deliberately made to differ by a small amount, known as the hysteresis H, to prevent noise (small deviations in temperature, due to various reasons) from switching the heater rapidly and unnecessarily on and off when the temperature is near the set-point. The fluctuations produced in the oven temperature are often significantly larger than the hysteresis values due to overshoot created by lags within the system. A typical temperature trace (the 'green' line in FIGURE 2) produced when the set point has a step change (the 'red' line) would look similar to that shown in the diagram (FIGURE 2). The 'blue' line shows the approximate heater output associated with the oven.

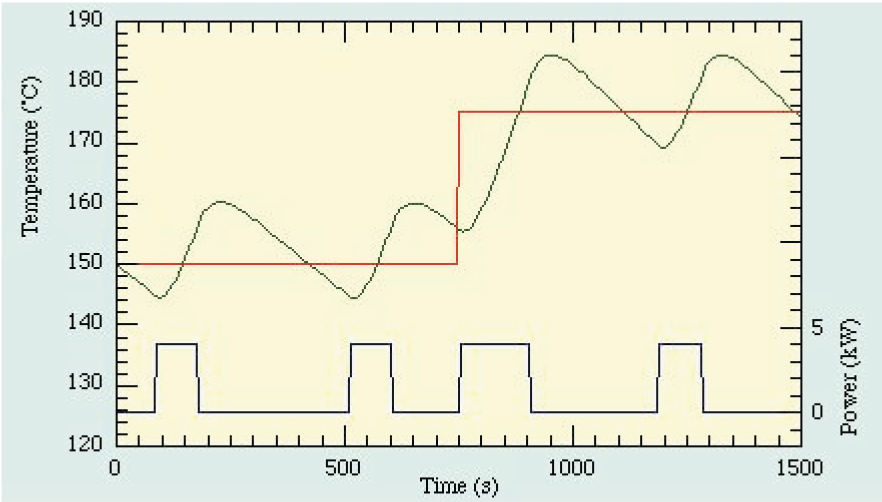


FIG. 2

Oven Temperature Controller Simulation

The above oven system can be modelled and simulated. FIGURE 3 shows one simulation actually produced. Refer to this as you read on.

The parameters used in the simulation which we will be concerned with are shown below and relate to the oven example shown in FIGURE 1 and the subsequent discussion.

Parameter	***Description***
C_h	*Heat capacity of heating element*
C_o	*Heat capacity of oven*
R_o	*Thermal resistance between oven and environment*
R_{ho}	*Thermal resistance between heating element and oven*
T_e	*Temperature of environment outside oven*
T_s	*Set-point temperature which controller tries to achieve*
H	*Hysteresis band of the On-Off controller*
P	*Proportional gain of PID controller*
I	*Integral action level of PID controller*
D	*Deriviative action level of PID controller*
M	*Maximum heater power*
Sensor lag	*The thermometer time constant*

Temperature Controller Simulation

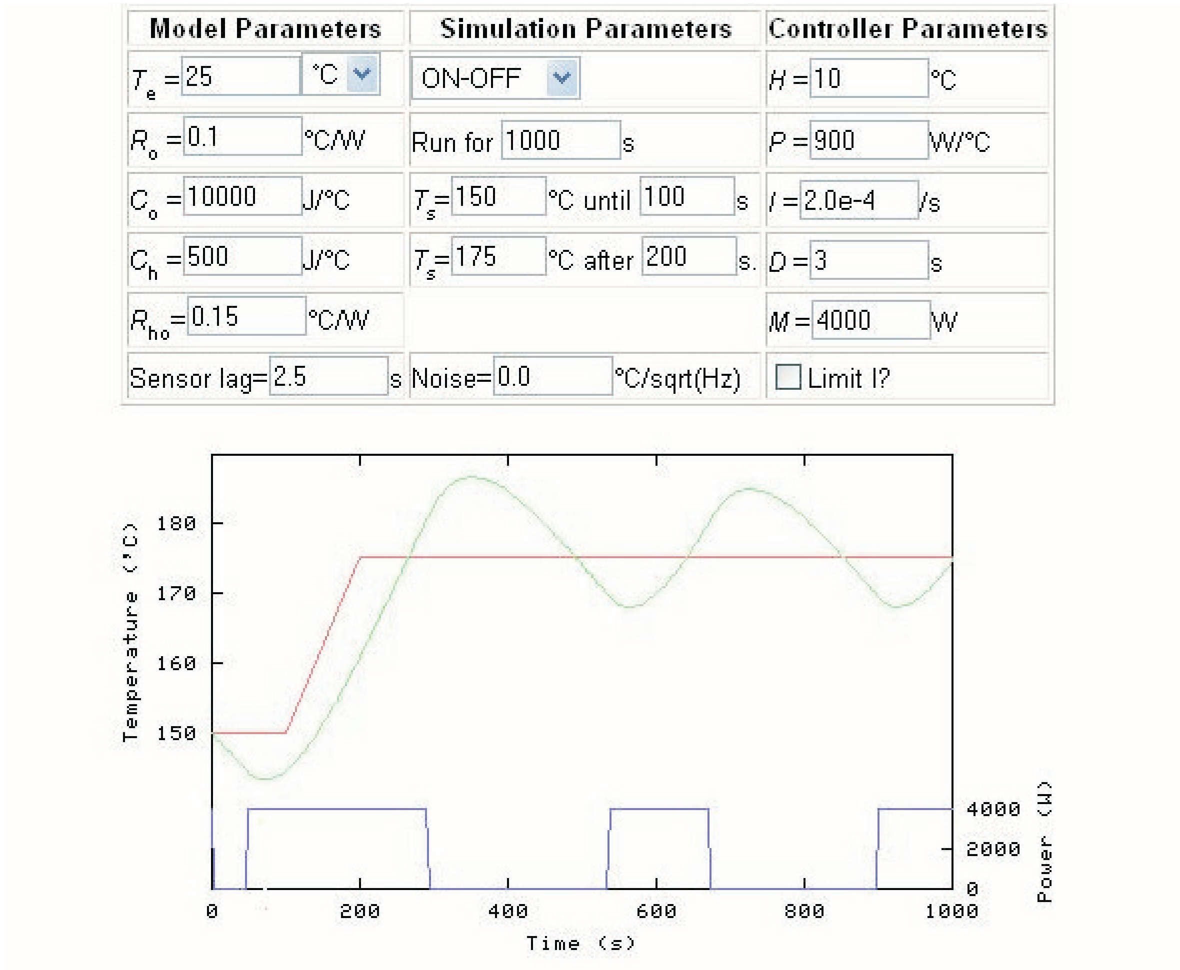


FIG. 3

The values of the **model parameters** in the boxes can be edited by selecting a box and typing in new values.

In the **Simulation Parameters**, the type of control can be changed using the drop down box menu to select either on-off (the control method shown in FIGURE 3) or PID. With the other simulation parameters the values can be changed by selecting a box and typing in new values. The values shown in FIGURE 3 are that the simulation will run for 1000s, the set-point temperature T_s will stay at 150°C until 100 s into the simulation when it will ramp up linearly to 175°C at 200 s.

In the **Control parameters** the values can be edited by selecting a box and typing in new values.

On the display the red line is the set-point temperature, T_s, the green line is the oven temperature, T_o, and the blue line is the actual heater power, M, being used by the oven (with on-off control this is the maximum power or zero power as the heater is on or off).

When values have been changed to the new ones to be simulated, clicking on the display gives the new result.

So for a simple change to PID control from on-off control, with all the other values remaining the same, the display will change to that shown in FIGURE 4.

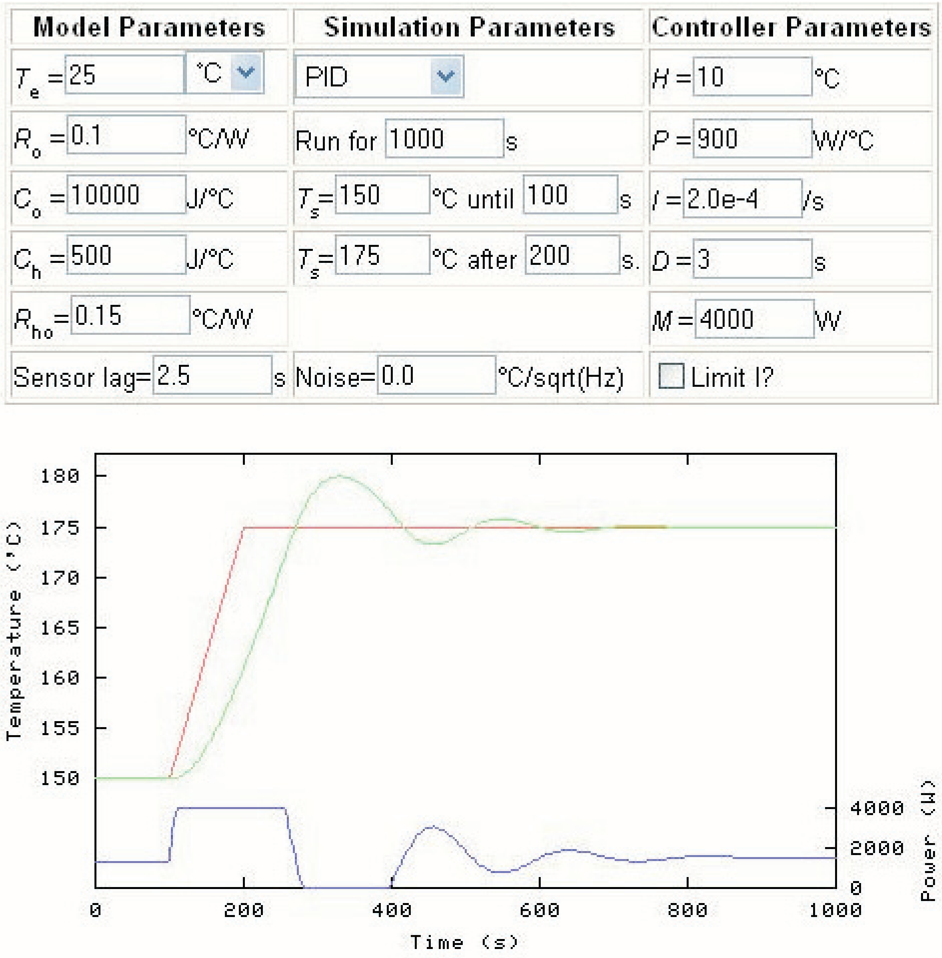


FIG. 4

Here you will notice that the control is much better than with on-off control, the actual temperature following the required set point temperature much more closely with only a small time lag and a few small fluctuations before quickly settling to its new required value. This indicates that the controller has been tuned to the oven settings beforehand and these values have been used in the simulation.

The effect of changing the value of the Integral action to zero (i.e. removing the integral action) can be determined.

Can you remember what the result of this should be from your previous studies?

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....

Without integral action offset is generally produced as shown in FIGURE 5.

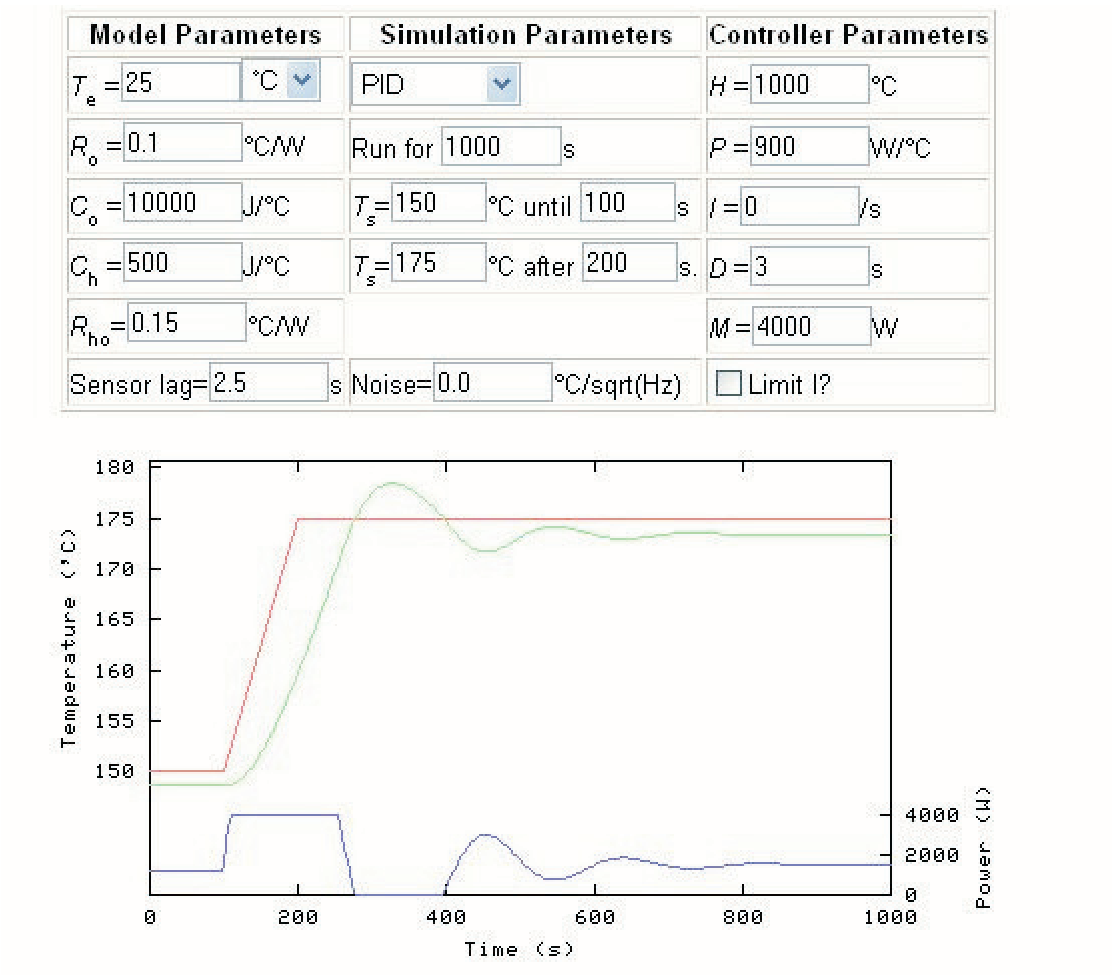


FIG. 5

The temperature trace is very similar to the previous one but the actual temperature, in this case, settles at a lower value than the set point.

Changing the integral action back to its original value 2.0e-4 and then changing the maximum power value, *M*, to 2000 watts gives the situation shown in FIGURE 6.

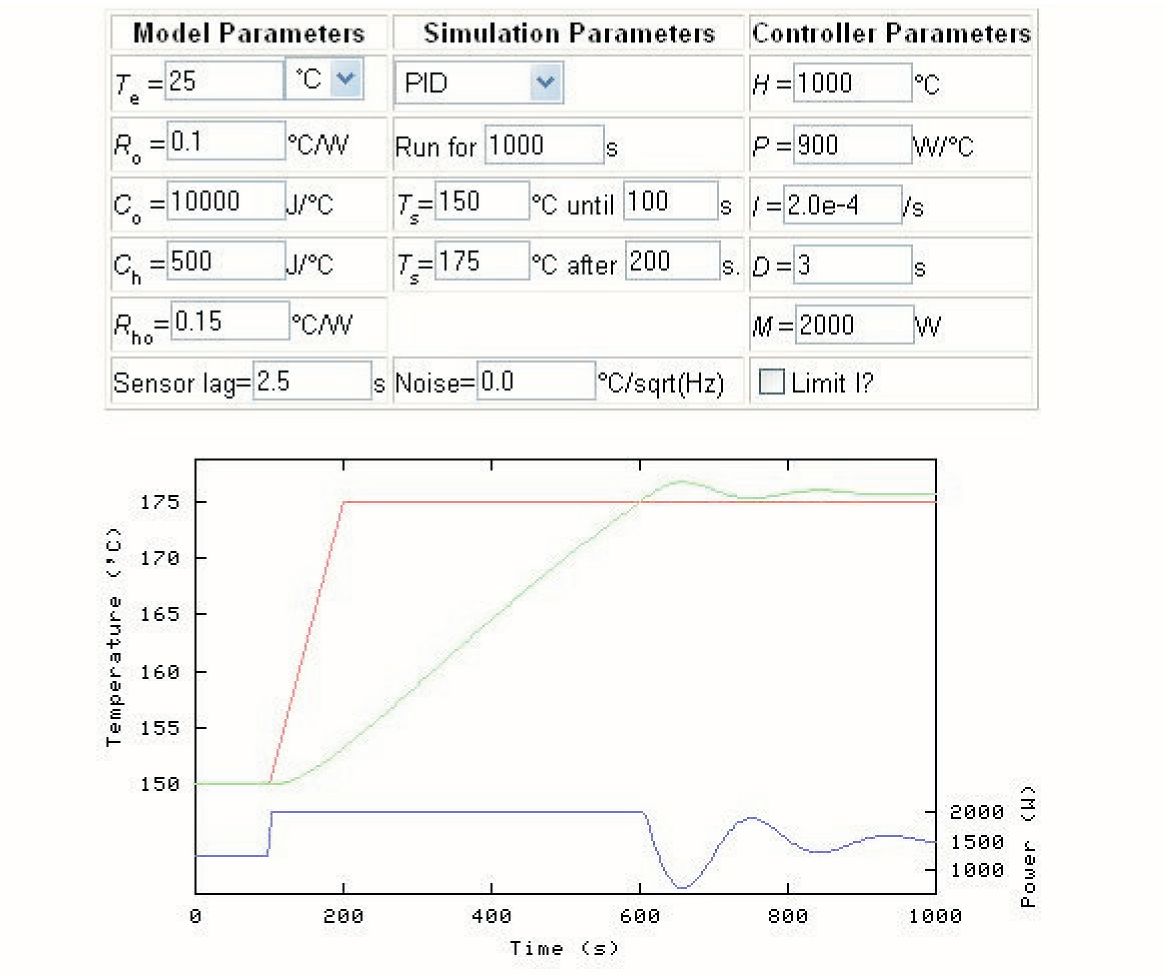


FIG. 6

Here the lower power input means that the heater takes longer to heat the oven to its new higher required temperature but overshoot is minimal. However, the controller is not tuned to the new conditions and has produced some offset. Also if we look at the power line, the heater seems to spend a lot of time close to its maximum output and would therefore appear to be undersized and may not be capable of reaching any higher temperature that may be required. So this simulation has shown that, although we could save capital costs, by using a smaller heater, we may find it is incapable of doing the job we want.

Increasing the heater size to 8000 watts produces, as shown in FIGURE 7, a much faster response but the temperature overshoot is much greater and it takes longer to settle at the new temperature with a small offset. If you look at the power curve you will see that the heater is vastly oversized as, apart from when the change in set point temperature occurs, the heater is only required to produce around 1500 watts. This bigger heater represents extra capital cost that could be avoided.

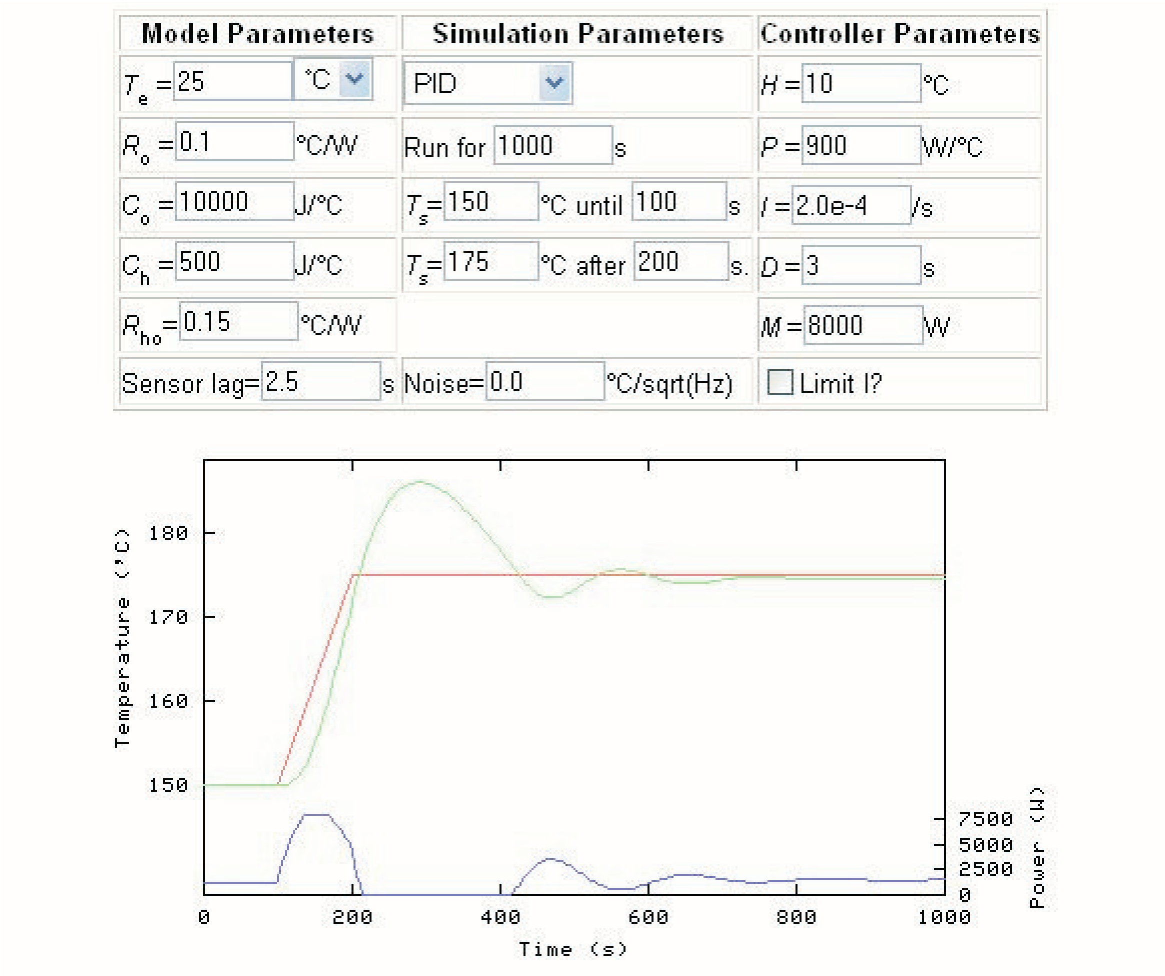


FIG. 7

From these simple examples you can see the usefulness of this simple simulation. You can change various parameters and see their effect without even needing to have the oven built. Thus the correct size of heating element to use, the values of control parameters, etc. can all be tested at minimal cost apart from those associated with developing the simulation. These can then be transferred to the design brief before going into production or before using the oven for a different set of conditions, e.g. we can change the set point or the heat capacity of the oven (simulating putting different or extra materials within the oven), etc., to observe their effect and see if the oven is suitable for the new duty before doing it in practice.

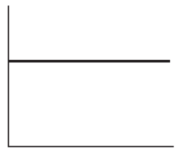
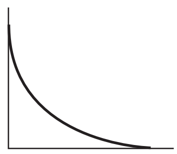
You should be able to extrapolate this usefulness to real industrial processes.

That completes our look at Mathematical modelling of processes and control systems. It has been a very brief examination and the topic is expanding rapidly as computer technology moves forward. It is now a degree subject in its own right and so our view is only meant to be an introduction. Specialists in simulation are now becoming a sought after commodity and the process is proving of real value and should be considered as part of any design or modification process regardless of the area being studied.

For now, complete the Self-Assessment Questions which follow to test your understanding of this lesson.

SELF-ASSESSMENT QUESTIONS

1. Complete the following table of Laplace Transforms.

	Symbol	*Graph*	*Laplace Transform F(s)*
Function of time f(t)	*Kf(t)*	*N/A*	
1st Derivative		*N/A*	$s\,F(s)\text{-}f(\text{-}0)$
Unit impulse	$\delta(t)$		*1*
Unit step	$u(t)$		
Ramp	t		$\frac{1}{s^2}$
Exponential decay			$\frac{1}{s + a}$
Exponential growth	$1 - e^{-at}$		$\frac{a}{s\,(s + a)}$
Sine wave	$Sin\ \omega t$		$\frac{\omega}{(s^2 + \omega^2)}$

2. (a) In the lesson we determined equations for the change in temperature detected by a thermocouple (ΔT_s) immersed in a fluid when:

   (i) a step change

   (ii) a ramp change of $0.5t$

   in temperature of the fluid (ΔT_f) was made.

   For both cases, calculate the change in temperature detected when the value of $\tau = 5$ and the time since the start of the change is 15 s.

   (b) Work out the value of $\dfrac{\Delta T_s}{\Delta T_f}$ after 15 s for both cases and explain why they are different.

3. Give **three** examples of different simulations you are familiar with and use these to determine two advantages and two disadvantages of the use of simulation.

ANSWERS TO SELF-ASSESSMENT QUESTIONS

1. See the table of Laplace transforms given in the lesson on page 8.

2. (a) (i) For a step change

$$\Delta T_s = \Delta T_f\left(1 - e^{-\frac{t}{\tau}}\right) = \Delta T_f\left(1 - e^{-\frac{15}{5}}\right)$$

$$= 0.95\Delta T_f$$

(ii) For the ramp change

$$\Delta T_s = 0.5\left[-\tau + t + \tau e^{-\frac{t}{\tau}}\right]$$

$$= 0.5\left[-5 + 15 + 5e^{-\frac{t}{\tau}}\right] = (0.5 \times 10.25)$$

$$= 5.125°\text{C}$$

(b) (i)

$$\frac{\Delta T_s}{\Delta T_f} = \frac{0.95\Delta T_f}{\Delta T_f} = 0.95$$

(ii) $\Delta T_{f,0} = 0$; $\Delta T_{f,t} = 0.5t = 0.5 \times 15 = 7.5°\text{C}$

$$\frac{\Delta T_s}{\Delta T_f} = \frac{5.125}{7.5} = 0.68$$

With the step input the thermocouple is gradually catching up with the change and will eventually reach it if given long enough (usually about 5 time constants is considered sufficient). So, in our case 15 seconds after the change (3 time constants), it is at 95% of the correct value.

With the ramped input, although the thermocouple is responding to the change, the change is continually occurring and so the thermocouple will always lag behind by a greater amount and is only at 68% of the correct value after the same time interval.

3. There are many possible answers here. Covered in the lesson were:

- Process simulations
- Computer games
- Kitchen designs.

These give rise to several advantages:

- Help improve techniques/designs
- Test possibilities
- Method of learning
- Can do them without the cost of real equipment.

Disadvantages are:

- Often not 'true to life', e.g. cannot predict the unexpected or cope with real situations that may occur
- Add to total cost and can be expensive the more accurate and “real” they are made to be
- May not recreate exact conditions.

SUMMARY

In this lesson we have looked at how Laplace transforms can be used as one method of solving differential equations which cannot be solved by normal integration.

A table of common transforms was given in the text and these were used to solve differential equations produced from the mathematical modelling of real processes, to obtain results for changes that occur within the process, which are representative of the real processes.

This technique (or others) can be used along with spreadsheets or other computer software to produce simulations of the real process which can vary from simple graphs of predicted results to 3D simulation (virtual reality) of the process.