$$\int \vec{j}\cdot d\vec{A} = -dq/dt,$$

where we put in a minus sign because, with our convention, $d\vec{A}$ is a little vector pointing *outwards*, so the integral represents net flow of charge *out* from the surface, equal to the rate of *decrease* of the enclosed total charge.

To summarize: if the local charge densities are changing in time, that is, if charge is piling up in or leaving some region, then $\int \vec{j}\cdot d\vec{A} \neq 0$ over a closed surface around that region. That implies that $\int \vec{j}\cdot d\vec{A}$ over one surface spanning the wire will be *different* from $\int \vec{j}\cdot d\vec{A}$ over another surface spanning the wire if these two surfaces together make up a closed surface enclosing a region containing a changing amount of charge.

The key to fixing this up is to realize that although $\int \vec{j}\cdot d\vec{A} = -dq/dt \neq 0$, it can be written as another surface integral over the same surface, using the *first* Maxwell equation, that is, the integral over a closed surface

$$\int \vec{E}\cdot d\vec{A} = q/\varepsilon_0$$

where q is the total charge in the volume enclosed by the surface.

By taking the time rate of change of both sides, we find

$$\frac{d}{dt}\int \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\frac{dq}{dt}$$

Putting this together with $\int \vec{j}\cdot d\vec{A} = -dq/dt$ gives:

$$\varepsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A} + \int \vec{j}\cdot d\vec{A} = 0 \qquad \text{(I)}$$

Why can we use that this is 0 in the original amperes law? Is it because we look at an instantaneous current and that it is 0?

for *any closed* surface, and consequently this is a surface integral that must be the same for *any* surface spanning the path or circuit! (Because two different surfaces spanning the same circuit add up to a closed surface. We'll ignore the technically trickier case where the two surfaces intersect each other, creating multiple volumes—there one must treat each created volume separately to get the signs right.)

Therefore, this is the way to generalize Ampere's law from the magnetostatic situation to the case where charge densities are varying with time, that is to say the path integral

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \int \left(\vec{j} + \varepsilon_0 \frac{d\vec{E}}{dt}\right)\cdot d\vec{A}$$

and this gives the same result for any surface spanning the path.

http://galileoandeinstein.physics.virginia.edu/more_stuff/Maxwell_Eq.html

What I wanted was an illustration for how the mathematical justification is right by pointing out where the dE/dt effects the magnetic field. Can you make that and relate it to (I) above?