The equation (334) is taken from the bottom here

http://farside.ph.utexas.edu/teaching/em/lectures/node38.html

The equation (337) is taken from the top here

http://farside.ph.utexas.edu/teaching/em/lectures/node39.html

These solutions can be recombined to form a single vector solution

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{j}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,d^3\mathbf{r}'. \tag{334}$$

The Biot-Savart law

According to Eq. (316), we can obtain an expression for the electric field generated by stationary charges by taking minus the gradient of Eq. (335). This yields

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \rho(\mathbf{r}')\,\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\,d^3\mathbf{r}', \tag{336}$$

which we recognize as Coulomb's law written for a continuous charge distribution. According to Eq. (318), we can obtain an equivalent expression for the magnetic field generated by steady currents by taking the curl of Eq. (334). This gives

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{j}(\mathbf{r}')\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\,d^3\mathbf{r}', \tag{337}$$

where use has been made of the vector identity $\nabla\times(\phi\mathbf{A}) = \phi\nabla\times\mathbf{A} + \nabla\phi\times\mathbf{A}$. Equation (337) is known as the *Biot-Savart law* after the French physicists Jean Baptiste Biot and Felix Savart: it completely specifies the magnetic field generated by a steady (but otherwise quite general) distributed current.

$$\nabla\times A = \frac{\mu_0}{4\pi}\int \nabla\times\frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}d^3\boldsymbol{r}'$$

$$= \frac{\mu_0}{4\pi}\int \nabla\times\frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}d^3\boldsymbol{r}'$$

$$\begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \dfrac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r_x}-\boldsymbol{r}'|} & \dfrac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r_y}-\boldsymbol{r}'|} & \dfrac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r_z}-\boldsymbol{r}'|} \end{vmatrix}$$

$$\left(\frac{\partial}{\partial y}\frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r_z}-\boldsymbol{r}'|} - \frac{\partial}{\partial z}\frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r_y}-\boldsymbol{r}'|}\right)i$$

$$-\left(\frac{\partial}{\partial x}\frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r_z}-\boldsymbol{r}'|} - \frac{\partial}{\partial z}\frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r_x}-\boldsymbol{r}'|}\right)j$$

$$+\left(\frac{\partial}{\partial x}\frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r_y}-\boldsymbol{r}'|} - \frac{\partial}{\partial y}\frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r_x}-\boldsymbol{r}'|}\right)k$$

$$\left(\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial y}\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}+\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}\frac{\partial}{\partial y}\boldsymbol{j}(\boldsymbol{r}')-\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial z}\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}-\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}\frac{\partial}{\partial z}\boldsymbol{j}(\boldsymbol{r}')\right)i$$

$$-\left(\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial x}\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}+\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}\frac{\partial}{\partial x}\boldsymbol{j}(\boldsymbol{r}')-\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial z}\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}-\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}\frac{\partial}{\partial z}\boldsymbol{j}(\boldsymbol{r}')\right)j$$

$$\left(\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial x}\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}+\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}\frac{\partial}{\partial x}\boldsymbol{j}(\boldsymbol{r}')-\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial y}\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}-\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}\frac{\partial}{\partial y}\boldsymbol{j}(\boldsymbol{r}')\right)k$$

$$\left(\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial y}\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}-\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial z}\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}+\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}\frac{\partial}{\partial y}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}\frac{\partial}{\partial z}\boldsymbol{j}(\boldsymbol{r}')\right)i$$

$$-\left(\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial x}\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}-\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial z}\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}+\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}\frac{\partial}{\partial x}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}\frac{\partial}{\partial z}\boldsymbol{j}(\boldsymbol{r}')\right)j$$

$$\left(\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial x}\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}-\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial y}\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}+\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}\frac{\partial}{\partial x}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}\frac{\partial}{\partial y}\boldsymbol{j}(\boldsymbol{r}')\right)k$$

$$\left(\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial y}\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}-\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial z}\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}\right)i$$

$$+\left(\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}\frac{\partial}{\partial y}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}\frac{\partial}{\partial z}\boldsymbol{j}(\boldsymbol{r}')\right)i$$

$$-\left(\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial x}\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}-\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial z}\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}\right)j$$

$$-\left(\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}\frac{\partial}{\partial x}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}\frac{\partial}{\partial z}\boldsymbol{j}(\boldsymbol{r}')\right)j$$

$$\left(\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial x}\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}-\boldsymbol{j}(\boldsymbol{r}')\frac{\partial}{\partial y}\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}\right)k$$

$$\left(\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}\frac{\partial}{\partial x}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r}_x-\boldsymbol{r}'|}\frac{\partial}{\partial y}\boldsymbol{j}(\boldsymbol{r}')\right)k$$

$$\boldsymbol{j}(\boldsymbol{r}')\left(\frac{\partial}{\partial y}\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}-\frac{\partial}{\partial z}\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}\right)i$$

$$+\left(\frac{1}{|\boldsymbol{r}_z-\boldsymbol{r}'|}\frac{\partial}{\partial y}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r}_y-\boldsymbol{r}'|}\frac{\partial}{\partial z}\boldsymbol{j}(\boldsymbol{r}')\right)i$$

$$-\boldsymbol{j}(\boldsymbol{r}')\left(\frac{\partial}{\partial x}\frac{1}{|\boldsymbol{r_z}-\boldsymbol{r}'|}-\frac{\partial}{\partial z}\frac{1}{|\boldsymbol{r_x}-\boldsymbol{r}'|}\right)j$$

$$-\left(\frac{1}{|\boldsymbol{r_z}-\boldsymbol{r}'|}\frac{\partial}{\partial x}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r_x}-\boldsymbol{r}'|}\frac{\partial}{\partial z}\boldsymbol{j}(\boldsymbol{r}')\right)j$$

$$+\boldsymbol{j}(\boldsymbol{r}')\left(\frac{\partial}{\partial x}\frac{1}{|\boldsymbol{r_y}-\boldsymbol{r}'|}-\frac{\partial}{\partial y}\frac{1}{|\boldsymbol{r_x}-\boldsymbol{r}'|}\right)k$$

$$\left(\frac{1}{|\boldsymbol{r_y}-\boldsymbol{r}'|}\frac{\partial}{\partial x}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r_x}-\boldsymbol{r}'|}\frac{\partial}{\partial y}\boldsymbol{j}(\boldsymbol{r}')\right)k$$

$$\boldsymbol{j}(\boldsymbol{r}')\left(\left(\frac{\partial}{\partial y}\frac{1}{|\boldsymbol{r_z}-\boldsymbol{r}'|}-\frac{\partial}{\partial z}\frac{1}{|\boldsymbol{r_y}-\boldsymbol{r}'|}\right)i-\left(\frac{\partial}{\partial x}\frac{1}{|\boldsymbol{r_z}-\boldsymbol{r}'|}-\frac{\partial}{\partial z}\frac{1}{|\boldsymbol{r_x}-\boldsymbol{r}'|}\right)j+\left(\frac{\partial}{\partial x}\frac{1}{|\boldsymbol{r_y}-\boldsymbol{r}'|}-\frac{\partial}{\partial y}\frac{1}{|\boldsymbol{r_x}-\boldsymbol{r}'|}\right)k\right)$$

$$+\left(\frac{1}{|\boldsymbol{r_z}-\boldsymbol{r}'|}\frac{\partial}{\partial y}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r_y}-\boldsymbol{r}'|}\frac{\partial}{\partial z}\boldsymbol{j}(\boldsymbol{r}')\right)i$$

$$-\left(\frac{1}{|\boldsymbol{r_z}-\boldsymbol{r}'|}\frac{\partial}{\partial x}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r_x}-\boldsymbol{r}'|}\frac{\partial}{\partial z}\boldsymbol{j}(\boldsymbol{r}')\right)j$$

$$\left(\frac{1}{|\boldsymbol{r_y}-\boldsymbol{r}'|}\frac{\partial}{\partial x}\boldsymbol{j}(\boldsymbol{r}')-\frac{1}{|\boldsymbol{r_x}-\boldsymbol{r}'|}\frac{\partial}{\partial y}\boldsymbol{j}(\boldsymbol{r}')\right)k$$

$$\boldsymbol{j}(\boldsymbol{r}')\left(\nabla x\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}\right)+$$

$$\nabla\boldsymbol{j}(\boldsymbol{r}')\times\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

I can't get the magnetic equation (337) from (334). I need a mathematical calculation from (334) to (337). In order for me to get this I need to have every step of the calculation done. And answer with pc signs if not I will not either get anything.

This was the identity I used

(8) Prove $\nabla x(\phi \mathbf{A}) = (\nabla \phi)x\mathrm{A} + \phi(\nabla x\mathbf{A})$

$$\nabla x(\phi A) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \phi A_1 & \phi A_2 & \phi A_3 \end{vmatrix}$$

$$= \left(\frac{\partial(\phi A_3)}{\partial y} - \frac{\partial(\phi A_2)}{\partial z}\right)i - \left(\frac{\partial(\phi A_3)}{\partial x} - \frac{\partial(\phi A_1)}{\partial z}\right)j + \left(\frac{\partial(\phi A_2)}{\partial x} - \frac{\partial(\phi A_1)}{\partial y}\right)k$$

$$= \left(\phi\frac{\partial A_3}{\partial y} + A_3\frac{\partial \phi}{\partial y} - \phi\frac{\partial A_2}{\partial y} - A_2\frac{\partial \phi}{\partial y}\right)i - \left(\phi\frac{\partial A_3}{\partial x} + A_3\frac{\partial \phi}{\partial x} - \phi\frac{\partial A_1}{\partial z} - A_1\frac{\partial \phi}{\partial z}\right)j + \left(\phi\frac{\partial A_2}{\partial x} + A_2\frac{\partial \phi}{\partial x} - \phi\frac{\partial A_1}{\partial y} - A_1\frac{\partial \phi}{\partial y}\right)k$$

$$= \left[\left(A_3\frac{\partial \phi}{\partial y} - A_2\frac{\partial \phi}{\partial y}\right)i - \left(A_3\frac{\partial \phi}{\partial x} - A_1\frac{\partial \phi}{\partial z}\right)j + \left(A_2\frac{\partial \phi}{\partial x} - A_1\frac{\partial \phi}{\partial y}\right)k\right]$$

$$+\left[\left(\phi\frac{\partial A_3}{\partial y} - \phi\frac{\partial A_2}{\partial y}\right)i - \left(\phi\frac{\partial A_3}{\partial x} - \phi\frac{\partial A_1}{\partial z}\right)j + \left(\phi\frac{\partial A_2}{\partial x} - \phi\frac{\partial A_1}{\partial y}\right)k\right]$$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix} + \phi\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$\therefore \quad \nabla x(\phi \mathbf{A}) = (\nabla \phi)x\mathrm{A} + \phi(\nabla x\mathbf{A})$