The Hilbert matrix $H_n = [h_{ij}]$ is the square matrix of size n whose entries are

$$h_{i,j} = \frac{1}{i+j-1} \qquad \text{for} \qquad 1 \le i, j \le n.$$

If you are using Maple or Mathematica make sure that entries in the Hilbert matrices are decimal approximations and not rational numbers, so $h_{i,j}$ should be entered as $1.0/(i+j-1)$. In Maple your program must set Digits:=14 before doing anything else. Whatever your programming language, all computations must be done in double precision.

1. For $n = 7$, 14, and 21, solve the system of equations $H_n x = b$, where b is the vector whose entries are given by

   $$b_i = \sum_{j=1}^{n} h_{i,j}.$$

   Use Gaussian elimination with scaled partial pivoting. Have your program print the solution, the final values of the NROW array, and the determinant of the transformed Hilbert matrix. (The closer this number is to 0, the closer H_n is to being singular and hence the greater the round-off error in the solution.)

2. Find the Doolittle LU decomposition of H_5 (the Hilbert matrix of size 5) with scaled partial pivoting. Use this decomposition to find the inverse of this matrix by solving 5 sets of linear equations. Have your program print the inverse matrix, the NROW array, the L and U matrices, and the product LU (which should be H_5 with its rows permuted).