http://www.columbia.edu/~ks20/4404-Sigman/4404-Notes-ITM.pdf

$f(x;\lambda) = \lambda e^{-\lambda x}$.

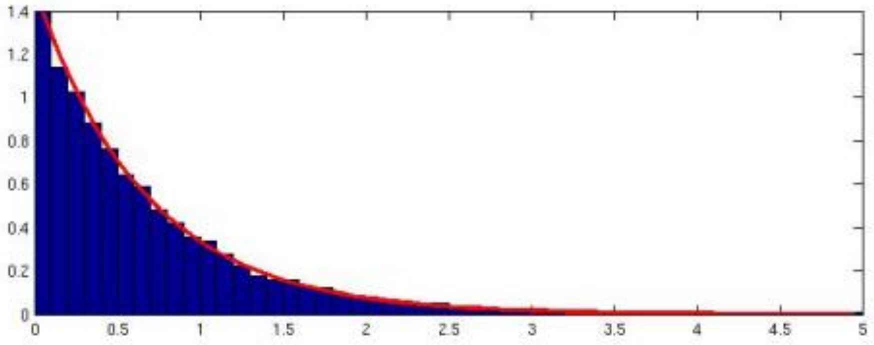
$F(x;\lambda) = 1 - e^{-\lambda x}$ og $F^{-1}(u) = -\lambda^{-1}\log(1-u)$.

Generate

$u \sim \text{Uniform}[0,1]$.

$x = -\lambda^{-1}\log(1-u)$

We simulate the x's and get



We see that the pdf of F, f has the same shape as the simulated x's. In general this seems to be the approach

We have the cdf, F:

$$F(x)$$

We find the inverse:

$$F^{-1}$$

We simulate the x's from the inverse and get that

$$F^{-1} = f(x)$$

I need a mathematical proof for this? For why this is approximately right. I only want a mathematical proof and if it is an approximation please a proof for why the approximation holds. The link above explains this theorem in general I believe. I specifically need to emphasize the part of the proof for why this manipulation of F gives values that is along the pdf, f, of cdf, F because this is my main problem.

1.1 Examples

The inverse transform method can be used in practice as long as we are able to get an explicit formula for $F^{-1}(y)$ in closed form. We illustrate with some examples. We use the notation $U \sim unif(0,1)$ to denote that U is a rv with the continuous uniform distribution over the interval $(0,1)$.

1. *Exponential distribution:* $F(x) = 1 - e^{-\lambda x}$, $x \geq 0$, where $\lambda > 0$ is a constant. Solving the equation $y = 1 - e^{-\lambda x}$ for x in terms of $y \in (0,1)$ yields $x = F^{-1}(y) = -(1/\lambda)\ln(1-y)$. This yields $X = -(1/\lambda)\ln(1-U)$. But (as is easily checked) $1 - U \sim unif(0,1)$ since $U \sim unif(0,1)$ and thus we can simplify the algorithm by replacing $1-U$ by U: