The causal solution$ f(x, t)$ of the one-dimensional diffusion equation with source term *f*



 (1)

is given by



 (2)

where $G(x, t; x^{'}, t')$ is the causal Green function satisfying



 (3)

a) Change variables in (3) to $τ=t-t'$, $z = x-x’$. Letting $G(z, τ)= g(z, τ)H(τ)$ with $H$ the Heaviside step function[[1]](#footnote-1), show (3) is solved if $g$ satisfies



 (4)

Solve by Fourier transforming with respect to the position variable. Hence obtain an expression for $G\left(x, t; x^{'}, t^{'}\right).$

b) The temperature $Φ(x, t)$ of a long uniform insulating metal rod satisfies



A heating element is used to create an initial temperature distribution $Φ\left(x, 0\right)=$ $T\_{0}e^{-λx^{2}}.$ Writing $ϕ\left(x, t\right)=Φ\left(x, t\right)H(t)$ obtain an equation for $ϕ$, and combining (1), (2) and the answer to a) identify the temperature variation at the center of the rod, $Φ(0, t)$.

1. $H\left(x\right)=1$ for $x>0$, =1/2 for $x=0$ and $=0$ for$ x<0$ and satisfies $dH(x)/dx = δ(x).$ [↑](#footnote-ref-1)