Below R stands for the field of real numbers, C stands for the field of complex numbers.

1. Let T be a linear transformation from the set $P_2(R)$ of all polynomials of degree at most 2 into itself, $T : P_2(R) \to P_2(R)$, given by

$$T(f) = f' - f'', \qquad f \in P_2(R),$$

where f' is the first andf'' is the second derivative of f.

   (a) Find the null space $N(T)$ of T. What is the dimension of $N(T)$.

   (b) Find the matrix representation $[T]_\varepsilon$ of T in the basis below

$$\varepsilon = \{e_1(x) = 1, e_2(x) = x, e_3(x) = x^2\}.$$

2. Let $A = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix}$

   (a) Find a characteristic polynomial $f(t) = \det(A - tI)$.

   (b) Find the eigenvalues and corresponding eigenvectors of A.

   (c) Is A diagonalizable? Justify your answer by citing a proper theorem.

   (d) Find an invertible matrix Q such that $D = Q^{-1}AQ$ is diagonal.

3. Let $w_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ and $w_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \qquad w_1, w_2 \in R^3.$

   (a) Use Gram-Schmidt process to orthogonalize, and then to orthonormalize, the vectors w_1, w_2.

   (b) Let $W = \operatorname{span}\{w_1, w_2\}$. Find the orthogonal projection of $y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ onto W.