

1. If air resistance is proportional to the square of the instantaneous velocity, then the velocity  $v$  of a mass  $m$  dropped from a given height is determined from

$$m \frac{dv}{dt} = mg - kv^2, \quad k > 0.$$

Let:

$$v(0) = 0, \quad k = 0.125, \quad m = 5 \text{ slugs}, \quad \text{and} \quad g = 32 \text{ ft/s}^2.$$

- (a) Use a fourth-order Runge-Kutta (RK4) method with  $h = 1$  to approximate the velocity  $v(5)$ .
  - (b) Use Excel to graph the solution of the IVP on the interval  $[0, 6]$ .
  - (c) Use separation of variables to solve the IVP and then find the actual value.
2. Find the analytic solution of the initial-value problem (IVP)
- $$y' = -y + 10 \sin(3x), \quad y(0) = 0, \quad \text{on the interval } [0, 2].$$
- (a) Graph the solution and find its positive roots.
  - (b) Use Runge-Kutta (RK4) method with  $h = 0.1$  to approximate a solution of the initial-value problem (IVP)  $y' = -y + 10 \sin(3x), \quad y(0) = 0$ .