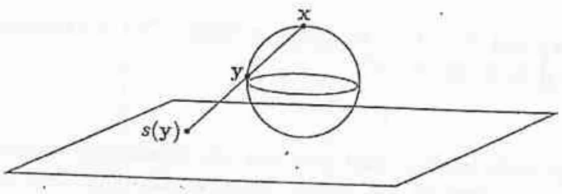


**Fig. 2.16.** Stereographic projection

▷ **Exercise 2.26.** Describe what stereographic projection does to

(1) the equator
(2) a longitudinal line from the North Pole to the South Pole
(3) a triangle drawn on the punctured sphere.

Note that we have shown that two spaces, one bounded and one unbounded, may be topologically equivalent. We wish to identify properties which homeomorphic spaces will share.

**(2.22) Definition.** *Property $P$ is a topological property if, whenever set $A$ has property $P$ and set $B$ is topologically equivalent to $A$, then $B$ also has property $P$.*

Thus, boundedness is not a topological property.

## 2.4 Compact sets

Although boundedness fails to be a topological property, a related notion does qualify:

**(2.23) Definition.** *A set $A$ is (sequentially) compact if every infinite sequence of points in $A$ has a limit point in $A$. That is, if $\{\mathbf{x}_i\}_{i=1}^{\infty}$ is a sequence and $\mathbf{x}_i \in A$ for each $i$, then there is a point $\mathbf{x} \in A$ so that $\mathbf{x}$ is a limit point of $\{\mathbf{x}_i\}_{i=1}^{\infty}$.*

There is another type of compactness, which will be briefly described in Chapter 3. Note that Definition 2.23 contains two claims: first, that every sequence has (at least one) limit point, and second, that this limit point will actually lie inside the set.