Problem D

Using direct tensor notation (i.e., without resorting to component forms) prove that if

$$\underline{u}, \underline{v}, \underline{w} \in \mathcal{V}_3 \qquad \& \qquad \alpha, \beta \in \mathbb{R}$$

a) $(\alpha \underline{u} \; \beta \underline{v}) = \alpha\beta(\underline{u}\,\underline{v})$

b) $(\underline{u}(\underline{v} + \underline{w})) = (\underline{u}\,\underline{v}) + (\underline{u}\,\underline{w})$

Hint: Take each expression and operate on a vector $\underline{a} \in \mathcal{V}_3$ and proceed accordingly. This is often a useful strategy.

Problem E

Using direct tensor notation, prove

$$\underline{u}\,\underset{\sim}{A} \cdot \underset{\sim}{B}\,\underline{v} = \underline{u} \cdot \underset{\sim}{A}\,(\underset{\sim}{B}\,\underline{v})$$

Problem F

A tensor $\underset{\sim}{T}$ transforms every vector into its mirror image with respect to the plane whose normal is

$$\underline{n} = \frac{\sqrt{2}}{2}(\underline{e}_1 + \underline{e}_2)$$

(a) Find the components of $\underset{\sim}{T}$ w.r.t the basis $\{\underline{e}_k\}$.

(b) Use this tensor to find the mirror image of

$$\underline{a} = \underline{e}_1 + 2\,\underline{e}_2$$