

Let  $h \in C^2(\mathbb{R}^3)$  be harmonic ( $\Delta h = 0$ ). Use Green's identity for  $\iint \Delta h \, d\text{vol}$  to show that  $\frac{1}{4\pi R^2} \iint h(x) \, d\text{area} = \frac{1}{4\pi} \iint_{|x|=1} h(Rx) \, d\text{area}$  is independent of the value of  $R$ . Then deduce the mean value theorem

$$h(0) = \frac{1}{4\pi R^2} \iint_{|x|=R} h(x) \, d\text{area}$$

Now what can you say if  $\lim_{x \rightarrow \infty} h(x) = 0$ ?

Since this is an analysis problem, please be sure to be rigorous, and include as much detail as possible so that I can understand. Please also state if you are making use of some fact or theorem. Thanks!