B.3 Graphically analyze the following problem:

$$\text{Maximize profit} = \$4X_1 + \$6X_2$$
$$\text{Subject to:} \quad 1X_1 + 2X_2 \le 8$$
$$6X_1 + 4X_2 \le 24$$

a) What is the optimal solution?
b) If the first constraint is altered to $1X_1 + 3X_2 \le 8$, does the feasible region or optimal solution change?

B.4 Consider the following linear programming problem:

$$\text{Maximize} \quad Z = 30X_1 + 10X_2$$
$$\text{Subject to:} \quad 3X_1 + X_2 \le 300$$
$$X_1 + X_2 \le 200$$
$$X_1 \le 100$$
$$X_2 \ge 50$$
$$X_1 - X_2 \le 0$$
$$X_1, X_2 \ge 0$$

a) Solve the problem graphically.
b) Is there more than one optimal solution? Explain.

B.5 Solve the following LP problem graphically:

$$\text{Minimize} \quad Z = 24X + 15Y$$
$$\text{Subject to:} \quad 7X + 11Y \ge 77$$
$$16X + 4Y \ge 80$$
$$X, Y \ge 0$$

B.6 Ed Silver Dog Food Company wishes to introduce a new brand of dog biscuits composed of chicken- and liver-flavored biscuits that meet certain nutritional requirements. The liver-flavored biscuits contain 1 unit of nutrient A and 2 units of nutrient B; the chicken-flavored biscuits contain 1 unit of nutrient A and 4 units of nutrient B. According to federal requirements, there must be at least 40 units of nutrient A and 60 units of nutrient B in a package of the new mix. In addition, the company has decided that there can be no more than 15 liver-flavored biscuits in a package. If it costs 1¢ to make 1 liver-flavored biscuit and 2¢ to make 1 chicken-flavored, what is the optimal product mix for a package of the biscuits to minimize the firm's cost?

a) Formulate this as a linear programming problem.
b) Solve this problem graphically, giving the optimal values of all variables.
c) What is the total cost of a package of dog biscuits using the optimal mix?

B.7 The Electrocomp Corporation manufactures two electrical products: air conditioners and large fans. The assembly process for each is similar in that both require a certain amount of wiring and drilling. Each air conditioner takes 3 hours of wiring and 2 hours of drilling. Each fan must go through 2 hours of wiring and 1 hour of drilling. During the next production period, 240 hours of wiring time are available and up to 140 hours of drilling time may be used. Each air conditioner sold yields a profit of $25. Each fan assembled may be sold for a $15 profit.

Formulate and solve this LP production-mix situation, and find the best combination of air conditioners and fans that yields the highest profit.

B.8 The Lauren Shur Tub Company manufactures two lines of bathtubs, called model A and model B. Every tub requires blending a certain amount of steel and zinc; the company has available a total of 25,000 lb of steel and 6,000 lb of zinc. Each model A bathtub requires a mixture of 125 lb of steel and 20 lb of zinc, and each yields a profit of $90. Each model B tub requires 100 lb of steel and 30 lb of zinc and can be sold for a profit of $70.

Find by graphical linear programming the best production mix of bathtubs.

B.9 The Grand Valley Company, run by the J. Motwani family, produces two products: bed mattresses and box springs. A prior contract requires that the firm produce at least 30 mattresses or box springs, in any combination, per week. In addition, union labor agreements demand that stitching machines be kept running at least 40 hours per week, which is one production period. Each box spring takes 2 hours of stitching time, and each mattress takes 1 hour on the machine. Each mattress produced costs $20; each box spring costs $24.

a) Formulate this problem so as to minimize total production costs.
b) Solve graphically.