

Let  $V$  be a  $\mathbb{C}$ -space of all complex valued polynomials with an inner product

$$\langle p, q \rangle = \int_0^1 p(t)\overline{q(t)} dt.$$

- (i) Let  $p$  be a polynomial and let  $M_p: V \rightarrow V$  be a linear operator that is given by  $M_p(q) := p \cdot q$ . Show that operator  $M_p$  has an adjoint and find it.
- (ii) Let  $D: V \rightarrow V$  be a linear operator that maps every polynomial in its derivative, by other words  $D(p) = p'$ . Show that  $D$  has not an adjoint.