From a Poisson (θ) distribution, a random sample $X_1, X_2, \ldots, X_n$ is selected. Given that $S = X_1 + \ldots + X_n$ is sufficient for θ and also has the Poisson($n\theta$) distribution we can define $g_{r,k}(s)$ by

$$g_{r,k}(s) = \{s!/(s-r)!\}\, n^{-r} \{1-(k/n)\}^{s-r},\ s = r, r+1, \ldots,\ 0 \text{ otherwise},$$

in which $r = 0, 1, 2, \ldots$ and k is a real constant.

(i) Show that $E[g_{r,k}(S)] = \theta^r \exp(-k\theta) = \tau_{r,k}(\theta)$.

(ii) Demonstrate that for all (r, k) values, the estimators $g_{r,k}(s)$ have minimum variance in the class of unbiased estimators of $\tau_{r,k}(\theta)$.

In the course of finding an estimator that is linearly related to the efficient score, find the (r, k) value for which an minimum vcariance bound (MVB) estimator exists.

For all other (r, k), the variance of the estimator exceeds the MVB.

Explain why is this so?