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Each question is equally weighted. Choose the single best answer for each question. Do not give multiple answers for any question.

1. First find a general solution of the differential equation  $\frac{dy}{dx} = 3y$ . Then find a particular solution that satisfies the initial condition that  $y(1) = 4$ .
2. Solve the initial value problem  $\frac{dy}{dx} = y^3$ ,  $y(0) = 1$ .
3. Nyobia had a population of 3 million in 1985. Assume that this country's population is growing continuously at a 5% annual rate and that Nyobia absorbs 40,000 newcomers per year. What will its population be in the year 2015?
4. Find the center and radius of the circle described in the equation  $2x^2 + 2y^2 - 6x + 2y = 3$ .
5. Find an equation of the ellipse with center  $(-2, 1)$ , horizontal major axis 10, and eccentricity  $\frac{2}{5}$ .
6. Determine whether or not the sequence  $a_n = \sqrt[n]{2^{2n+3}}$  converges and find its limit if it does converge.
7. Write the Taylor series with center zero for the function  $f(x) = \ln(1+x^2)$ .
8. Given  $a = 2i + 3j$ ,  $b = 3i + 5j$ , and  $c = 8i + 11j$  express  $c$  in the form  $ra + sb$  where  $r$  and  $s$  are scalars.
9. Given  $a = \langle 4, -3, -1 \rangle$  and  $b = \langle 1, 4, 6 \rangle$ , find  $a \times b$ .
10. Find the arc length of the curve given by  $x = \cos 3t$ ,  $y = \sin 3t$ ,  $z = 4t$ , from  $t = 0$  to  $t = \frac{\pi}{2}$ .
11. Compute the first-order partial derivatives of  $f(x, y) = \frac{2x}{x-y}$ .

12. Use the method of Lagrange multipliers to find the extreme values of  $3x - 4y + 12z$  on the spherical surface with equation  $x^2 + y^2 + z^2 = 1$ .
13. Evaluate  $\int_1^3 \int_0^1 (2x - 3y) dx dy$ .
14. Find the volume of the solid bounded by  $x = 0$ ,  $y = 0$ ,  $z = 0$ , and  $x + 2y + 3z = 6$  by triple integration.
15. Calculate the divergence and curl of the vector field  $F(x, y, z) = 2xi + 3yj + 4zk$ .
16. Use Green's theorem to evaluate  $\oint_C Pdx + Qdy$   
 $P(x, y) = xy$ ,  $Q(x, y) = e^x$ ;  $C$  is the curve that goes from  $(0, 0)$  to  $(2, 0)$  along the  $x$ -axis and then returns to  $(0, 0)$  along the parabola  $y = 2x - x^2$ .