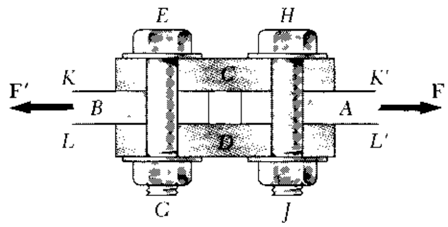


Fig. 1.20

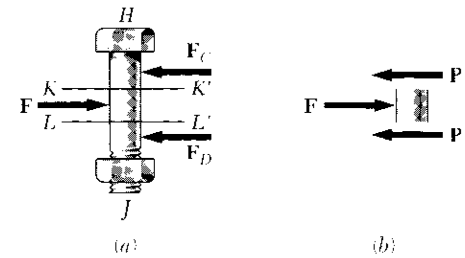


Fig. 1.21

The bolt we have just considered is said to be in *single shear*. Different loading situations may arise, however. For example, if splice plates C and D are used to connect plates A and B (Fig. 1.20), shear will take place in bolt HJ in each of the two planes KK' and LL' (and similarly in bolt EG). The bolts are said to be in *double shear*. To determine the average shearing stress in each plane, we draw free-body diagrams of bolt HJ and of the portion of bolt located between the two planes (Fig. 1.21). Observing that the shear P in each of the sections is $P = F/2$, we conclude that the average shearing stress is

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A} \tag{1.10}$$

1.7. BEARING STRESS IN CONNECTIONS

Bolts, pins, and rivets create stresses in the members they connect, along the *bearing surface*, or surface of contact. For example, consider again the two plates A and B connected by a bolt CD that we have discussed in the preceding section (Fig. 1.18). The bolt exerts on plate A a force **P** equal and opposite to the force **F** exerted by the plate on the bolt (Fig. 1.22). The force **P** represents the resultant of elementary forces distributed on the inside surface of a half-cylinder of diameter d and of length t equal to the thickness of the plate. Since the distribution of these forces—and of the corresponding stresses—is quite complicated, one uses in practice an average nominal value σ_b of the stress, called the *bearing stress*, obtained by dividing the load P by the area of the rectangle representing the projection of the bolt on the plate section (Fig. 1.23). Since this area is equal to td, where t is the plate thickness and d the diameter of the bolt, we have

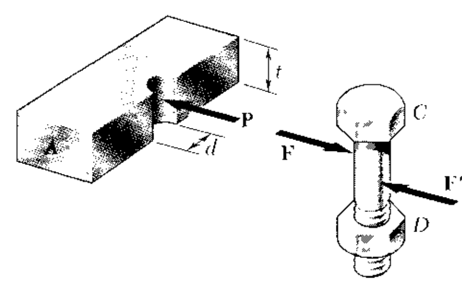


Fig. 1.22

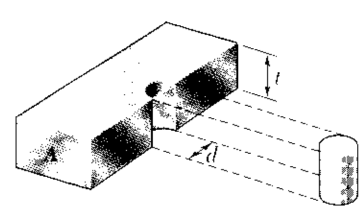


Fig. 1.23

$$\sigma_b = \frac{P}{A} = \frac{P}{td} \tag{1.11}$$

1.8. APPLICATION TO THE ANALYSIS AND DESIGN OF SIMPLE STRUCTURES

We are now in a position to determine the stresses in the members and connections of various simple two-dimensional structures and, thus, to design such structures.

As an example, let us return to the structure of Fig. 1.1 that we have already considered in Sec. 1.2 and let us specify the supports and connections at A, B, and C. As shown in Fig. 1.24, the 20-mm-diameter rod BC has flat ends of 20×40-mm rectangular cross section, while boom AB has a 30×50-mm rectangular cross section and is fitted with a clevis at end B. Both members are connected at B by a pin from which the 30-kN load is suspended by means of a U-shaped bracket. Boom AB is supported at A by a pin fitted into a double bracket, while rod BC is connected at C to a single bracket. All pins are 25 mm in diameter.

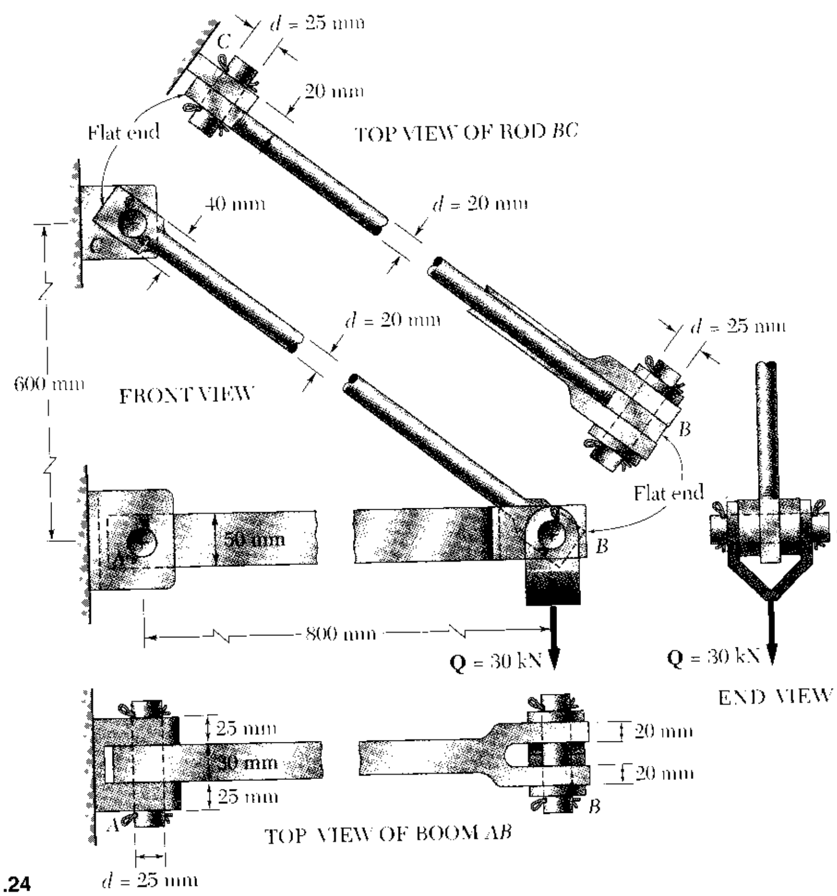


Fig. 1.24

a. Determination of the Normal Stress in Boom *AB* and Rod *BC*. As we found in Secs. 1.2 and 1.4, the force in rod BC is $F_{BC} = 50$ kN (tension) and the area of its circular cross section is $A = 314 \times 10^{-6}\ \text{m}^2$; the corresponding average normal stress is $\sigma_{BC} = +159$ MPa. However, the flat parts of the rod are also under tension and at the narrowest section, where a hole is located, we have

$$A = (20\text{ mm})(40\text{ mm} - 25\text{ mm}) = 300 \times 10^{-6}\text{ m}^2$$

The corresponding average value of the stress, therefore, is

$$(\sigma_{BC})_{\text{end}} = \frac{P}{A} = \frac{50 \times 10^3\text{ N}}{300 \times 10^{-6}\text{ m}^2} = 167\text{ MPa}$$

Note that this is an *average value*; close to the hole, the stress will actually reach a much larger value, as you will see in Sec. 2.18. It is clear that, under an increasing load, the rod will fail near one of the holes rather than in its cylindrical portion; its design, therefore, could be improved by increasing the width or the thickness of the flat ends of the rod.

Turning now our attention to boom AB, we recall from Sec. 1.2 that the force in the boom is $F_{AB} = 40\text{ kN}$ (compression). Since the area of the boom's rectangular cross section is $A = 30\text{ mm} \times 50\text{ mm} = 1.5 \times 10^{-3}\text{ m}^2$, the average value of the normal stress in the main part of the rod, between pins A and B, is

$$\sigma_{AB} = -\frac{40 \times 10^3\text{ N}}{1.5 \times 10^{-3}\text{ m}^2} = -26.7 \times 10^6\text{ Pa} = -26.7\text{ MPa}$$

Note that the sections of minimum area at A and B are not under stress, since the boom is in compression, and, therefore, *pushes* on the pins (instead of *pulling* on the pins as rod BC does).

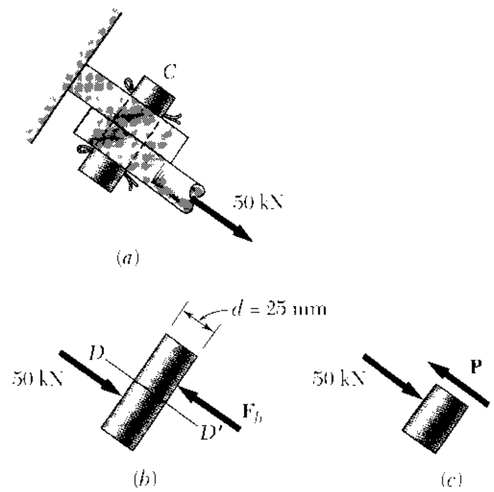


Fig. 1.25

b. Determination of the Shearing Stress in Various Connections. To determine the shearing stress in a connection such as a bolt, pin, or rivet, we first clearly show the forces exerted by the various members it connects. Thus, in the case of pin C of our example (Fig. 1.25*a*), we draw Fig. 1.25*b*, showing the 50-kN force exerted by member BC on the pin, and the equal and opposite force exerted by the bracket. Drawing now the diagram of the portion of the pin located below the plane DD' where shearing stresses occur (Fig. 1.25*c*), we conclude that the shear in that plane is $P = 50\text{ kN}$. Since the cross-sectional area of the pin is

$$A = \pi r^2 = \pi\left(\frac{25\text{ mm}}{2}\right)^2 = \pi(12.5 \times 10^{-3}\text{ m})^2 = 491 \times 10^{-6}\text{ m}^2$$

we find that the average value of the shearing stress in the pin at C is

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{50 \times 10^3\text{ N}}{491 \times 10^{-6}\text{ m}^2} = 102\text{ MPa}$$

Considering now the pin at A (Fig. 1.26), we note that it is in double shear. Drawing the free-body diagrams of the pin and of the portion of pin located between the planes DD' and EE' where shearing stresses occur, we conclude that $P = 20\text{ kN}$ and that

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{20\text{ kN}}{491 \times 10^{-6}\text{ m}^2} = 40.7\text{ MPa}$$

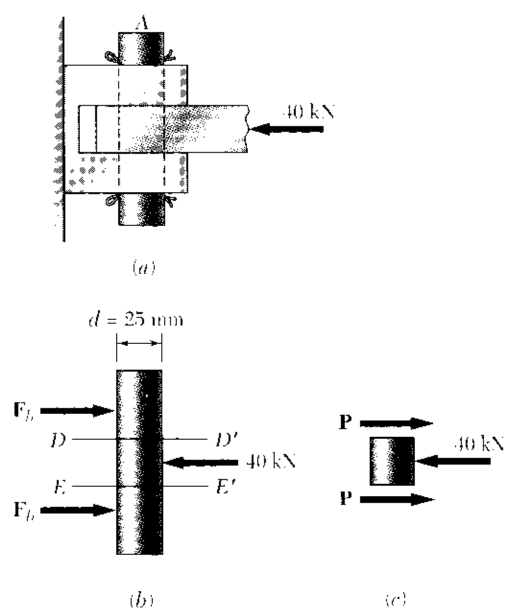


Fig. 1.26

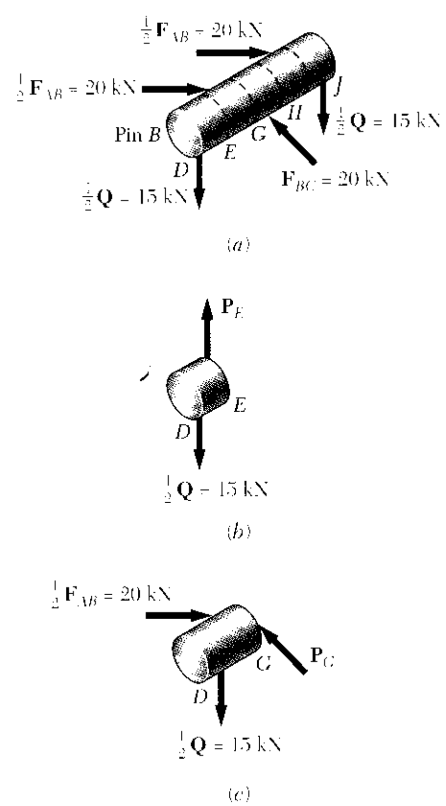


Fig. 1.27

Considering the pin at B (Fig. 1.27*a*), we note that the pin may be divided into five portions which are acted upon by forces exerted by the boom, rod, and bracket. Considering successively the portions DE (Fig. 1.27*b*) and DG (Fig. 1.27*c*), we conclude that the shear in section E is $P_E = 15$ kN, while the shear in section G is $P_G = 25$ kN. Since the loading of the pin is symmetric, we conclude that the maximum value of the shear in pin B is $P_G = 25$ kN, and that the largest shearing stresses occur in sections G and H, where

$$\tau_{\text{ave}} = \frac{P_G}{A} = \frac{25\text{ kN}}{491 \times 10^{-6}\text{ m}^2} = 50.9\text{ MPa}$$

c. Determination of the Bearing Stresses. To determine the nominal bearing stress at A in member AB, we use formula (1.11) of Sec. 1.7. From Fig. 1.24, we have $t = 30$ mm and $d = 25$ mm. Recalling that $P = F_{AB} = 40$ kN, we have

$$\sigma_b = \frac{P}{td} = \frac{40\text{ kN}}{(30\text{ mm})(25\text{ mm})} = 53.3\text{ MPa}$$

To obtain the bearing stress in the bracket at A, we use $t = 2(25\text{ mm}) = 50$ mm and $d = 25$ mm:

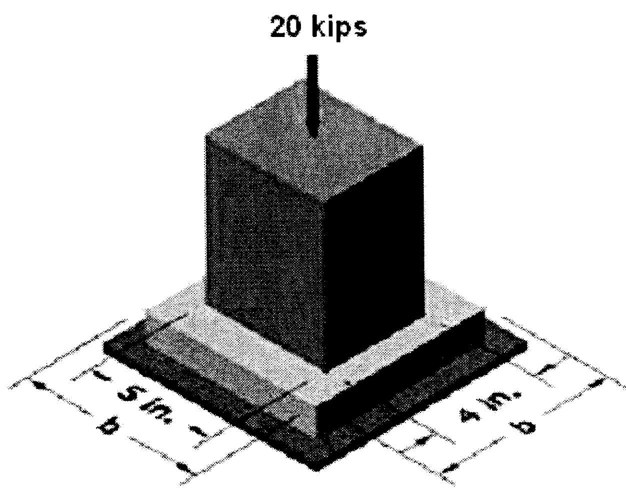
$$\sigma_b = \frac{P}{td} = \frac{40\text{ kN}}{(50\text{ mm})(25\text{ mm})} = 32.0\text{ MPa}$$

The bearing stresses at B in member AB, at B and C in member BC, and in the bracket at C are found in a similar way.

1.9. METHOD OF PROBLEM SOLUTION

You should approach a problem in mechanics of materials as you would approach an actual engineering situation. By drawing on your own experience and intuition, you will find it easier to understand and formulate the problem. Once the problem has been clearly stated, however, there is no place in its solution for your particular fancy. Your solution must be based on the fundamental principles of statics and on the principles you will learn in this course. Every step you take must be justified on that basis, leaving no room for your "intuition." After an answer has been obtained, it should be checked. Here again, you may call upon your common sense and personal experience. If not completely satisfied with the result obtained, you should carefully check your formulation of the problem, the validity of the methods used in its solution, and the accuracy of your computations.

The *statement* of the problem should be clear and precise. It should contain the given data and indicate what information is required. A simplified drawing showing all essential quantities involved should be included. The solution of most of the problems you will encounter will necessitate that you first determine the *reactions at supports* and *inter-*


20 kips
5 in.
b
4 in.
b