$$\sin 3\alpha = 3\sin\alpha\cos^2\alpha - \sin^3\alpha = 3\sin\alpha - 4\sin^3\alpha$$
$$\cos 3\alpha = \cos^3\alpha - 3\cos\alpha\sin^2\alpha = 4\cos^3\alpha - 3\cos\alpha$$
$$\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha} \qquad \cot 3\alpha = \frac{\cot^3\alpha - 3\cot\alpha}{3\cot^2\alpha - 1}$$
$$\sin n\alpha = 2\sin(n-1)\alpha\cdot\cos\alpha - \sin(n-2)\alpha$$
$$\cos n\alpha = 2\cos(n-1)\alpha\cdot\cos\alpha - \cos(n-2)\alpha$$
$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$
$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} = 2\sin\frac{\beta+\alpha}{2}\sin\frac{\beta-\alpha}{2}$$
$$\tan\alpha + \tan\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} \qquad \tan\alpha - \tan\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$$
$$\cot\alpha + \cot\beta = \frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta} \qquad \cot\alpha - \cot\beta = \frac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$$
$$1 - \cos\alpha = 2\sin^2\frac{\alpha}{2} \qquad 1 + \cos\alpha = 2\cos^2\frac{\alpha}{2}$$
$$1 + \sin\alpha = \sin\frac{\pi}{2} + \sin\alpha = 2\sin\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$$
$$1 - \sin\alpha = \sin\frac{\pi}{2} + \sin\alpha = 2\sin\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\cos\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$$
$$1 + \tan\alpha = \tan\frac{\pi}{4} + \tan\alpha = \sqrt{2}\,\frac{\sin(\pi/4 + \alpha)}{\cos\alpha}$$
$$1 - \tan\alpha = \tan\frac{\pi}{4} - \tan\alpha = \sqrt{2}\,\frac{\sin(\pi/4 - \alpha)}{\cos\alpha}$$

NOTES FOR PROBLEMS 10-15.

In finding the solutions to trigonometric equations, it is often necessary to find values for sin and cos with angles greater than 180 degrees. For example, consider $\sin^{-1}(-.5)$. A calculator will return a negative value for this: $-30°$. Call the absolute value for this alpha. There will be two solutions in the range $180°$ to $360°$. One solution is a $360° - 30°$ and the other is at $180° + 30°$. This holds true for all the negative values of $\sin\theta$.

For the cosine, consider all the angles in the range from $0°$ to $360°$ for which the cosine is -0.5. A calculator will return a value of $120°$, which is one of the solutions we seek. If we call this angle alpha, there will be another solution at $360°$ minus alpha, or $360° - 120° = 240°$.