TABLE 1 EQUIVALENT EXPRESSIONS FOR SIN α, COS α, TAN α, AND COT α

Function	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$
$\sin\alpha$	$\sin\alpha$	$\pm\sqrt{1-\cos^2\alpha}$	$\dfrac{\tan\alpha}{\pm\sqrt{1+\tan^2\alpha}}$	$\dfrac{1}{\pm\sqrt{1+\cot^2\alpha}}$
$\cos\alpha$	$\pm\sqrt{1-\sin^2\alpha}$	$\cos\alpha$	$\dfrac{1}{\pm\sqrt{1+\tan^2\alpha}}$	$\dfrac{\cot\alpha}{\pm\sqrt{1+\cot^2\alpha}}$
$\tan\alpha$	$\dfrac{\sin\alpha}{\pm\sqrt{1-\sin^2\alpha}}$	$\dfrac{\pm\sqrt{1-\cos^2\alpha}}{\cos\alpha}$	$\tan\alpha$	$\dfrac{1}{\cot\alpha}$
$\cot\alpha$	$\dfrac{\pm\sqrt{1-\sin^2\alpha}}{\sin\alpha}$	$\dfrac{\cos\alpha}{\pm\sqrt{1-\cos^2\alpha}}$	$\dfrac{1}{\tan\alpha}$	$\cot\alpha$

TABLE 2

$$\sin\alpha = \frac{1}{\csc\alpha} \qquad \cos\alpha = \frac{1}{\sec\alpha} \qquad \tan\alpha = \frac{1}{\cot\alpha} = \frac{\sin\alpha}{\cos\alpha}$$

$$\csc\alpha = \frac{1}{\sin\alpha} \qquad \sec\alpha = \frac{1}{\cos\alpha} \qquad \cot\alpha = \frac{1}{\tan\alpha} = \frac{\cos\alpha}{\sin\alpha}$$

$$\sin^2\alpha + \cos^2\alpha = 1 \qquad 1 + \tan^2\alpha = \sec^2\alpha \qquad 1 + \cot^2\alpha = \csc^2\alpha$$

$$\sin^2\alpha = \tfrac{1}{2}(1 - \cos 2\alpha) \qquad \cos^2\alpha = \tfrac{1}{2}(1 + \cos 2\alpha)$$

$$\sin^3\alpha = \tfrac{1}{4}(3\sin\alpha - \sin 3\alpha) \qquad \cos^3\alpha = \tfrac{1}{4}(\cos 3\alpha + 3\cos\alpha)$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$

$$\cos\alpha\cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$

$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha + \beta) + \tfrac{1}{2}\sin(\alpha - \beta)$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} = \frac{1}{2}\left(\sqrt{1+\sin\alpha} - \sqrt{1-\sin\alpha}\right)$$

positive if $\frac{\alpha}{2}$ in quadrants I or II, negative otherwise

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}} = \frac{1}{2}\left(\sqrt{1+\sin\alpha} + \sqrt{1-\sin\alpha}\right)$$

positive if $\frac{\alpha}{2}$ in I or IV, negative otherwise.

$$\tan\frac{\alpha}{2} = \frac{-1 \pm \sqrt{1+\tan^2\alpha}}{\tan\alpha}$$

$$= \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

positive if $\frac{\alpha}{2}$ in I or III, negative otherwise.

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \pm 2\sin\alpha\sqrt{1-\sin^2\alpha} = \pm 2\cos\alpha\sqrt{1-\cos^2\alpha}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$