12.1 Exercises

1. Suppose you start at the origin, move along the x-axis a distance of 4 units in the positive direction, and then move downward a distance of 3 units. What are the coordinates of your position?

2. Sketch the points $(0, 5, 2)$, $(4, 0, -1)$, $(2, 4, 6)$, and $(1, -1, 2)$ on a single set of coordinate axes.

3. Which of the points $P(6, 2, 3)$, $Q(-5, -1, 4)$, and $R(0, 3, 8)$ is closest to the xz-plane? Which point lies in the yz-plane?

4. What are the projections of the point $(2, 3, 5)$ on the xy-, yz-, and xz-planes? Draw a rectangular box with the origin and $(2, 3, 5)$ as opposite vertices and with its faces parallel to the coordinate planes. Label all vertices of the box. Find the length of the diagonal of the box.

5. Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $x + y = 2$.

6. (a) What does the equation $x = 4$ represent in $\mathbb{R}^2$? What does it represent in $\mathbb{R}^3$? Illustrate with sketches.
   (b) What does the equation $y = 3$ represent in $\mathbb{R}^3$? What does $z = 5$ represent? What does the pair of equations $y = 3$, $z = 5$ represent? In other words, describe the set of points (x, y, z) such that $y = 3$ and $z = 5$. Illustrate with a sketch.

7. Show that the triangle with vertices $P(-2, 4, 0)$, $Q(1, 2, -1)$, and $R(-1, 1, 2)$ is an equilateral triangle.

8. Find the lengths of the sides of the triangle with vertices $A(1, 2, -3)$, $B(3, 4, -2)$, and $C(3, -2, 1)$. Is ABC a right triangle? Is it an isosceles triangle?

9. Determine whether the points lie on a straight line.
   (a) $A(5, 1, 3)$, $B(7, 9, -1)$, $C(1, -15, 11)$
   (b) $K(0, 3, -4)$, $L(1, 2, -2)$, $M(3, 0, 1)$

10. Find the distance from $(3, 7, -5)$ to each of the following.
    (a) The xy-plane
    (b) The yz-plane
    (c) The xz-plane
    (d) The x-axis
    (e) The y-axis
    (f) The z-axis

11. Find an equation of the sphere with center $(1, -4, 3)$ and radius 5. What is the intersection of this sphere with the xz-plane?

12. Find an equation of the sphere with center $(6, 5, -2)$ and radius $\sqrt{7}$. Describe its intersection with each of the coordinate planes.

13. Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has center $(3, 8, 1)$.

14. Find an equation of the sphere that passes through the origin and whose center is $(1, 2, 3)$.

15–18 Show that the equation represents a sphere, and find its center and radius.

15. $x^2 + y^2 + z^2 - 6x + 4y - 2z = 11$

16. $x^2 + y^2 + z^2 = 4x - 2y$

17. $x^2 + y^2 + z^2 = x + y + z$

18. $4x^2 + 4y^2 + 4z^2 - 8x + 16y = 1$

19. (a) Prove that the midpoint of the line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$
    (b) Find the lengths of the medians of the triangle with vertices $A(1, 2, 3)$, $B(-2, 0, 5)$, and $C(4, 1, 5)$.

20. Find an equation of a sphere if one of its diameters has endpoints $(2, 1, 4)$ and $(4, 3, 10)$.

21. Find equations of the spheres with center $(2, -3, 6)$ that touch (a) the xy-plane, (b) the yz-plane, (c) the xz-plane.

22. Find an equation of the largest sphere with center $(5, 4, 9)$ that is contained in the first octant.

23–34 Describe in words the region of $\mathbb{R}^3$ represented by the equation or inequality.

23. $y = -4$

24. $x = 10$

25. $x > 3$

26. $y \geqslant 0$

27. $0 \leqslant z \leqslant 6$

28. $y = z$

29. $x^2 + y^2 + z^2 > 1$

30. $1 \leqslant x^2 + y^2 + z^2 \leqslant 25$

31. $x^2 + y^2 + z^2 - 2z < 3$

32. $x^2 + y^2 = 1$

33. $x^2 + z^2 \leqslant 9$

34. $xyz = 0$

35–38 Write inequalities to describe the region.

35. The half-space consisting of all points to the left of the xz-plane

36. The solid rectangular box in the first octant bounded by the planes $x = 1$, $y = 2$, and $z = 3$

37. The region consisting of all points between (but not on) the spheres of radius r and R centered at the origin, where $r < R$

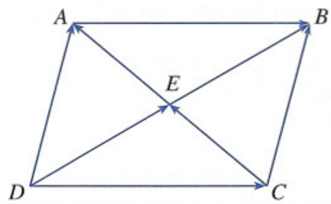
38. The solid upper hemisphere of the sphere of radius 2 centered at the origin

12.2 Exercises

1. Are the following quantities vectors or scalars? Explain.
(a) The cost of a theater ticket
(b) The current in a river
(c) The initial flight path from Houston to Dallas
(d) The population of the world

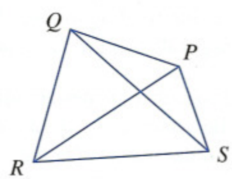
2. What is the relationship between the point (4, 7) and the vector $\langle 4, 7 \rangle$? Illustrate with a sketch.

3. Name all the equal vectors in the parallelogram shown.



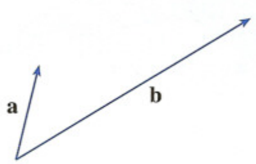
4. Write each combination of vectors as a single vector.

(a) $\overrightarrow{PQ} + \overrightarrow{QR}$ (b) $\overrightarrow{RP} + \overrightarrow{PS}$
(c) $\overrightarrow{QS} - \overrightarrow{PS}$ (d) $\overrightarrow{RS} + \overrightarrow{SP} + \overrightarrow{PQ}$



5. Copy the vectors in the figure and use them to draw the following vectors.
(a) $\mathbf{u} + \mathbf{v}$ (b) $\mathbf{u} - \mathbf{v}$
(c) $\mathbf{v} + \mathbf{w}$ (d) $\mathbf{w} + \mathbf{v} + \mathbf{u}$

6. Copy the vectors in the figure and use them to draw the following vectors.
(a) $\mathbf{a} + \mathbf{b}$ (b) $\mathbf{a} - \mathbf{b}$
(c) $2\mathbf{a}$ (d) $-\frac{1}{2}\mathbf{b}$
(e) $2\mathbf{a} + \mathbf{b}$ (f) $\mathbf{b} - 3\mathbf{a}$



7–12 Find a vector $\mathbf{a}$ with representation given by the directed line segment $\overrightarrow{AB}$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

√**7.** $A(2, 3)$, $B(-2, 1)$ **8.** $A(-2, -2)$, $B(5, 3)$

9. $A(-1, -1)$, $B(-3, 4)$ **10.** $A(-2, 2)$, $B(3, 0)$

11. $A(0, 3, 1)$, $B(2, 3, -1)$ **12.** $A(4, 0, -2)$, $B(4, 2, 1)$

13–16 Find the sum of the given vectors and illustrate geometrically.

13. $\langle 3, -1 \rangle$, $\langle -2, 4 \rangle$ **14.** $\langle -2, -1 \rangle$, $\langle 5, 7 \rangle$

15. $\langle 0, 1, 2 \rangle$, $\langle 0, 0, -3 \rangle$ **16.** $\langle -1, 0, 2 \rangle$, $\langle 0, 4, 0 \rangle$

17–22 Find $|\mathbf{a}|$, $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, $2\mathbf{a}$, and $3\mathbf{a} + 4\mathbf{b}$.

17. $\mathbf{a} = \langle -4, 3 \rangle$, $\mathbf{b} = \langle 6, 2 \rangle$

18. $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{b} = \mathbf{i} + 5\mathbf{j}$

19. $\mathbf{a} = \langle 6, 2, 3 \rangle$, $\mathbf{b} = \langle -1, 5, -2 \rangle$

20. $\mathbf{a} = \langle -3, -4, -1 \rangle$, $\mathbf{b} = \langle 6, 2, -3 \rangle$

21. $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{j} + 2\mathbf{k}$

22. $\mathbf{a} = 3\mathbf{i} - 2\mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$

23–25 Find a unit vector that has the same direction as the given vector.

23. $\langle 9, -5 \rangle$ **24.** $12\mathbf{i} - 5\mathbf{j}$

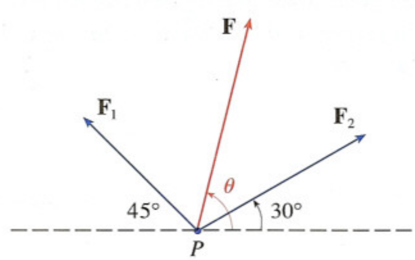
25. $8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

26. Find a vector that has the same direction as $\langle -2, 4, 2 \rangle$ but has length 6.

27. If $\mathbf{v}$ lies in the first quadrant and makes an angle $\pi/3$ with the positive x-axis and $|\mathbf{v}| = 4$, find $\mathbf{v}$ in component form.

28. If a child pulls a sled through the snow with a force of 50 N exerted at an angle of 38° above the horizontal, find the horizontal and vertical components of the force.

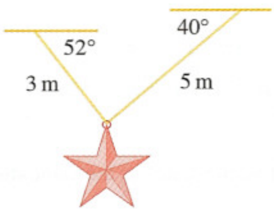
29. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ with magnitudes 10 lb and 12 lb act on an object at a point P as shown in the figure. Find the resultant force $\mathbf{F}$ acting at P as well as its magnitude and its direction. (Indicate the direction by finding the angle θ shown in the figure.)



30. Velocities have both direction and magnitude and thus are vectors. The magnitude of a velocity vector is called *speed.* Suppose that a wind is blowing from the direction N45°W at a speed of 50 km/h. (This means that the direction from which the wind blows is 45° west of the northerly direction.) A pilot is steering a plane in the direction N60°E at an airspeed (speed in still air) of 250 km/h. The *true course,* or *track,* of the plane is the direction of the resultant of the velocity vectors of the plane and the wind. The *ground speed* of the plane is the magnitude of the resultant. Find the true course and the ground speed of the plane.

√ **31.** A woman walks due west on the deck of a ship at 3 mi/h. The ship is moving north at a speed of 22 mi/h. Find the speed and direction of the woman relative to the surface of the water.

32. Ropes 3 m and 5 m in length are fastened to a holiday decoration that is suspended over a town square. The decoration has a mass of 5 kg. The ropes, fastened at different heights, make angles of 52° and 40° with the horizontal. Find the tension in each wire and the magnitude of each tension.



33. A clothesline is tied between two poles, 8 m apart. The line is quite taut and has negligible sag. When a wet shirt with a mass of 0.8 kg is hung at the middle of the line, the midpoint is pulled down 8 cm. Find the tension in each half of the clothesline.

34. The tension $\mathbf{T}$ at each end of the chain has magnitude 25 N. What is the weight of the chain?

35. If A, B, and C are the vertices of a triangle, find $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$.

36. Let C be the point on the line segment AB that is twice as far from B as it is from A. If $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$, and $\mathbf{c} = \overrightarrow{OC}$, show that $\mathbf{c} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$.

√ **37.** (a) Draw the vectors $\mathbf{a} = \langle 3, 2 \rangle$, $\mathbf{b} = \langle 2, -1 \rangle$, and $\mathbf{c} = \langle 7, 1 \rangle$.
(b) Show, by means of a sketch, that there are scalars s and t such that $\mathbf{c} = s\mathbf{a} + t\mathbf{b}$.
(c) Use the sketch to estimate the values of s and t.
(d) Find the exact values of s and t.

38. Suppose that $\mathbf{a}$ and $\mathbf{b}$ are nonzero vectors that are not parallel and $\mathbf{c}$ is any vector in the plane determined by $\mathbf{a}$ and $\mathbf{b}$. Give a geometric argument to show that $\mathbf{c}$ can be written as $\mathbf{c} = s\mathbf{a} + t\mathbf{b}$ for suitable scalars s and t. Then give an argument using components.

√ **39.** If $\mathbf{r} = \langle x, y, z \rangle$ and $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$, describe the set of all points (x, y, z) such that $|\mathbf{r} - \mathbf{r}_0| = 1$.

40. If $\mathbf{r} = \langle x, y \rangle$, $\mathbf{r}_1 = \langle x_1, y_1 \rangle$, and $\mathbf{r}_2 = \langle x_2, y_2 \rangle$, describe the set of all points (x, y) such that $|\mathbf{r} - \mathbf{r}_1| + |\mathbf{r} - \mathbf{r}_2| = k$, where $k > |\mathbf{r}_1 - \mathbf{r}_2|$.

41. Figure 16 gives a geometric demonstration of Property 2 of vectors. Use components to give an algebraic proof of this fact for the case $n = 2$.

42. Prove Property 5 of vectors algebraically for the case $n = 3$. Then use similar triangles to give a geometric proof.

43. Use vectors to prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

44. Suppose the three coordinate planes are all mirrored and a light ray given by the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ first strikes the xz-plane, as shown in the figure. Use the fact that the angle of incidence equals the angle of reflection to show that the direction of the reflected ray is given by $\mathbf{b} = \langle a_1, -a_2, a_3 \rangle$. Deduce that, after being reflected by all three mutually perpendicular mirrors, the resulting ray is parallel to the initial ray. (American space scientists used this principle, together with laser beams and an array of corner mirrors on the Moon, to calculate very precisely the distance from the Earth to the Moon.)