Consider the map $f : (x, y) \to (x + y, xy)$ for $0 < y < x$. Find the inverse

$$f^{-1} : (\xi, \eta) = (x + y, xy) \to (x, y)$$

Compute $\frac{\partial f^{-1}}{\partial(\xi,\eta)}$ and $\frac{\partial f}{\partial(x,y)}$, and confirm the rule

$$\frac{\partial f^{-1}}{\partial(\xi, \eta)} = (\frac{\partial f}{\partial(x, y)})^{-1} \circ f^{-1}(\xi, \eta)$$

directly.

As this is an analysis question, please be sure to be rigorous and as detailed as possible. I would also prefer the solution in PDF format. Thank You.