1) An undamped oscillator has period $\tau_o = 1.000$s, but I now add a little damping so that its period changes to $\tau_1 = 1.001$s. What is the damping factor β? By what factor will the amplitude of oscillation decrease after 10 cycles? Which effect of damping would be more noticeable, the change of period or the decrease of the amplitude?

2) As the damping on an oscillator is increased there comes a point when the name "oscillator" seems barely appropriate. (a) Illustrate this, prove that a critically damped oscillator can never pass through the origin x = 0 more than once. (b) Prove the same for an overdamped oscillator.

3) The solution for x(t) for a driven, undamped oscillator is most conveniently found in the form:

$$x(t) = A\cos(\omega t - \delta) + e^{-\beta t}[B_1 \cos(\omega_1 t) + B_2 \sin(\omega_1 t)]$$

Solve that the equation and the corresponding expression for x dot, to give the coefficients B_1 and B_2 in terms of A, δ, and the initial position and velocity x_o and v_o. Verify the expressions given in:

$$B1 = x_o - \mathrm{A}\cos\delta \text{ and } B2 = \frac{1}{\omega_1}(v_o - \omega \mathrm{A}\sin\delta + \beta \mathrm{B}_1).$$

4) We know that if the driving frequency ω is varied, the maximum response (A^2) of a driven damped oscillator occurs at $\omega \approx \omega_0$ (if the natural frequency is ω_0 and the damping constant $\beta << \omega_0$). Show that A^2 is equal to half its maximum value when $\omega \approx \omega_0 +/- \beta$, so that the full width at half maximum is just 2β.