

2. Consider a Cable Of Unspecified Length Which Is Connected Between Two Fixed Points In the $x - z$ Coordinate System, At $(x, z) = (0, 1)$ And (x_0, z_0) , Where x Is a Horizontal Coordinate And z Is a Vertical Coordinate, Which Has a Mass Density Per Unit Length Of ρ , and is In a Gravitational Field Which Points In the Negative $\hat{z}$ Direction, With Gravitational Constant g . The Objective Is To Determine the Specific Shape Of the Cable In a 2D Sense, That Is, To Find $z(x)$ Which Minimizes the Potential Energy, U , Of the Cable. Recall That the Differential Potential Energy (With Reference To $U = 0$ At $z = 0$) Of a Differential Mass Element, dm , Is Given By $dU = glzdm$, Which Can Be Related To a Differential Length (Along the Cable) Element, ds , Where $dm = \rho l ds$, So That the Total Potential Energy Of the Cable Can Be Obtained As An Integral Along the Length Of the Cable, From the Point At $x = 0$ To the Point At $x = x_0$, Or $U = \rho gl^2 \int_{x=0}^{x=x_0} z ds = \int_0^{x_0} f(z, z_x, x) dx$. Here, It Should Be Noted That All Length Variables, x, z , Are Dimensionless, And Normalized With Respect To a Characteristic (Dimensional) Length, l . Also, Recall That the Extremum Of the Potential Energy Functional, $\delta U = 0$, Can Be Satisfied By the Solution Of the Euler Equation, $\frac{\partial f}{\partial z} - \frac{d}{dx} \left(\frac{\partial f}{\partial z_x} \right) = 0$, And If the Integrand Of the Functional Is Independent Of x , That Is, Of the Form $f(z, z_x)$, the Alternate Version Of the Euler Equation, $f - z_x \frac{\partial f}{\partial z_x} = K$, Can Be Used, Where the Constant