CHAPTER 14

Section 14.1

1.

a. We reject H_o if the calculated χ^2 value is greater than or equal to the tabled value of $\chi^2_{\alpha,k-1}$ from Table A.7. Since $12.25 \geq \chi^2_{.05,4} = 9.488$, we would reject H_o.

b. Since 8.54 is not $\geq \chi^2_{.01,3} = 11.344$, we would fail to reject H_o.

c. Since 4.36 is not $\geq \chi^2_{.10,2} = 4.605$, we would fail to reject H_o.

d. Since 10.20 is not $\geq \chi^2_{.01,5} = 15.085$, we would fail to reject H_o.

2.

a. In the d.f. = 2 row of Table A.7, our χ^2 value of 7.5 falls between $\chi^2_{.025,2} = 7.378$ and $\chi^2_{.01,2} = 9.210$, so the p-value is between .01 and .025, or .01 < p-value < .025.

b. With d.f. = 6, our χ^2 value of 13.00 falls between $\chi^2_{.05,6} = 12.592$ and $\chi^2_{.025,6} = 14.440$, so .025 < p-value < .05.

c. With d.f. = 9, our χ^2 value of 18.00 falls between $\chi^2_{.05,9} = 16.919$ and $\chi^2_{.025,9} = 19.022$, so .025 < p-value < .05.

d. With k = 5, d.f. = k – 1 = 4, and our χ^2 value of 21.3 exceeds $\chi^2_{.005,4} = 14.860$, so the p-value < .005.

e. The d.f. = k – 1 = 4 – 1 = 3; $\chi^2 = 5.0$ is less than $\chi^2_{.10,3} = 6.251$, so p-value > .10.

3. Using the number 1 for business, 2 for engineering, 3 for social science, and 4 for agriculture, let $p_i =$ the true proportion of all clients from discipline i. If the Statistics department's expectations are correct, then the relevant null hypothesis is $H_o : p_1 = .40, p_2 = .30, p_3 = .20, p_4 = .10,$ versus $H_a :$ The Statistics department's expectations are not correct. With d.f = k – 1 = 4 – 1 = 3, we reject H_o if $c^2 \geq c^2_{.05,3} = 7.815$. Using the proportions in H_o, the expected number of clients are :

Client's Discipline	Expected Number
Business	(120)(.40) = 48
Engineering	(120)(.30) = 36
Social Science	(120)(.20) = 24
Agriculture	(120)(.10) = 12

Since all the expected counts are at least 5, the chi-squared test can be used. The value of the test statistic is $c^2 = \sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i} = \sum_{allcells} \frac{(observed - \exp ected)^2}{\exp ected}$

$= \left[\frac{(52-48)^2}{48} + \frac{(38-36)^2}{36} + \frac{(21-24)^2}{24} + \frac{(9-12)^2}{12} \right] = 1.57$, which is not ≥ 7.815, so we fail to reject H_o. (Alternatively, p-value = $P(c^2 \geq 1.57)$ which is > .10, and since the p-value is not < .05, we reject H_o). Thus we have no evidence to suggest that the statistics department's expectations are incorrect.

4. The uniform hypothesis implies that $p_{i0} = \frac{1}{8} = .125$ for I = 1, …, 8, so $H_o : p_{10} = p_{20} = ... = p_{80} = .125$ will be rejected in favor of H_a if $c^2 \geq c^2_{.10,7} = 12.017$. Each expected count is np_{i0} = 120(.125) = 15, so $c^2 = \left[\frac{(12-15)^2}{15} + ... + \frac{(10-15)^2}{15} \right] = 4.80$. Because 4.80 is not ≥ 12.017 , we fail to reject H_o. There is not enough evidence to disprove the claim.

5. We will reject H_o if the p-value < .10. The observed values, expected values, and corresponding χ^2 terms are :

Obs	4	15	23	25	38	21	32	14	10	8
Exp	6.67	13.33	20	26.67	33.33	33.33	26.67	20	13.33	6.67
χ^2	1.069	.209	.450	.105	.654	.163	1.065	1.800	.832	.265

$\chi^2 = 1.069 + ... + .265 = 6.612$. With d.f. = 10 – 1 = 9, our χ^2 value of 6.612 is less than $\chi^2_{.10,9} = 14.684$, so the p-value > .10, which is not < .10, so we cannot reject H_o. There is no evidence that the data is not consistent with the previously determined proportions.

6. A 9:3:4 ratio implies that $p_{10} = \frac{9}{16} = .5625$, $p_{20} = \frac{3}{16} = .1875$, and $p_{30} = \frac{4}{16} = .2500$. With n = 195 + 73 + 100 = 368, the expected counts are 207.000, 69.000, and 92.000, so $\chi^2 = \left[\frac{(195-207)^2}{207} + \frac{(73-69)^2}{69} + \frac{(100-92)^2}{92} \right] = 1.623$. With d.f. = 3 – 1 = 2, our χ^2 value of 1.623 is less than $\chi^2_{.10,2} = 4.605$, so the p-value > .10, which is not < .05, so we cannot reject H_o. The data does confirm the 9:3:4 theory.

7. We test $H_o : p_1 = p_2 = p_3 = p_4 = .25$ vs. $H_a :$ at least one proportion $\neq .25$, and d.f. = 3. We will reject H_o if the p-value < .01.

Cell	1	2	3	4
Observed	328	334	372	327
Expected	340.25	340.25	340.25	34.025
χ^2 term	.4410	.1148	2.9627	.5160

$\chi^2 = 4.0345$, and with 3 d.f., p-value > .10, so we fail to reject H_o. The data fails to indicate a seasonal relationship with incidence of violent crime.

8. $H_o : p_1 = \frac{15}{365}, p_2 = \frac{46}{365}, p_3 = \frac{120}{365}, p_4 = \frac{184}{365},$ versus H_a : at least one proportion is not a stated in H_o. The degrees of freedom = 3, and the rejection region is $c^2 \ge c_{.01,3} = 11.344$.

Cell	1	2	3	4
Observed	11	24	69	96
Expected	8.22	25.21	65.75	100.82
c^2 term	.9402	.0581	.1606	.2304

$c^2 = \sum \frac{(obs - \exp)^2}{\exp} = 1.3893$, which is not ≥ 11.344, so H_o is not rejected. The data does not indicate a relationship between patients' admission date and birthday.

9.

a. Denoting the 5 intervals by $[0, c_1), [c_1, c_2), \ldots, [c_4, \infty)$, we wish c_1 for which $.2 = P(0 \le X \le c_1) = \int_0^{c_1} e^{-x} dx = 1 - e^{-c_1}$, so c_1 = -ln(.8) = .2231. Then $.2 = P(c_1 \le X \le c_2) \Rightarrow .4 = P(0 \le X_1 \le c_2) = 1 - e^{-c_2}$, so c_2 = -ln(.6) = .5108. Similarly, c_3 = -ln(.4) = .0163 and c_4 = -ln(.2) = 1.6094. the resulting intervals are [0, .2231), [.2231, .5108), [.5108, .9163), [.9163, 1.6094), and [1.6094, ∞).

b. Each expected cell count is 40(.2) = 8, and the observed cell counts are 6, 8, 10, 7, and 9, so $c^2 = \left[\frac{(6-8)^2}{8} + \ldots + \frac{(9-8)^2}{8}\right] = 1.25$. Because 1.25 is not $\ge c^2_{.10,4} = 7.779$, even at level .10 H_o cannot be rejected; the data is quite consistent with the specified exponential distribution.

10.

a. $c^2 = \sum_{i=1}^{k} \frac{(n_i - np_{i0})^2}{np_{i0}} = \sum_i \frac{N_i^2 - 2np_{i0}N_i + n^2 p_{i0}^2}{np_{i0}} = \sum_i \frac{N_i^2}{np_{i0}} - 2\sum_i N_i + n\sum_i p_{i0}$

$= \sum_i \frac{N_i^2}{np_{i0}} - 2n + n(1) = \sum_i \frac{N_i^2}{np_{i0}} - n$ as desired. This formula involves only one subtraction, and that at the end of the calculation, so it is analogous to the shortcut formula for s^2.

b. $c^2 = \frac{k}{n}\sum_i N_i^2 - n$. For the pigeon data, k = 8, n = 120, and $\Sigma N_i^2 = 1872$, so

$c^2 = \frac{8(1872)}{120} - 120 = 124.8 - 120 = 4.8$ as before.

11.

a. The six intervals must be symmetric about 0, so denote the 4th, 5th and 6th intervals by [0, a0, [a, b), [b, ∞). a must be such that $\Phi(a) = .6667\left(\frac{1}{2}+\frac{1}{6}\right)$, which from Table A.3 gives $a \approx .43$. Similarly $\Phi(b) = .8333$ implies $b \approx .97$, so the six intervals are ($-\infty$, -.97), [-.97, -.43), [-.43, 0), [0, .43), [.43, .97), and [.97, ∞).

b. The six intervals are symmetric about the mean of .5. From **a**, the fourth interval should extend from the mean to .43 standard deviations above the mean, i.e., from .5 to .5 + .43(.002), which gives [.5, .50086). Thus the third interval is [.5 - .00086, .5) = [.49914, .5). Similarly, the upper endpoint of the fifth interval is .5 + .97(.002) = .50194, and the lower endpoint of the second interval is .5 - .00194 = .49806. The resulting intervals are ($-\infty$, .49806), [.49806, .49914), [.49914, .5), [.5, .50086), [.50086, .50194), and [.50194, ∞).

c. Each expected count is $45\left(\frac{1}{6}\right) = 7.5$, and the observed counts are 13, 6, 6, 8, 7, and 5, so $c^2 = 5.53$. With 5 d.f., the p-value > .10, so we would fail to reject H_o at any of the usual levels of significance. There is no evidence to suggest that the bolt diameters are not normally distributed.

Section 14.2

12.

a. Let q denote the probability of a male (as opposed to female) birth under the binomial model. The four cell probabilities (corresponding to x = 0, 1, 2, 3) are $p_1(q) = (1-q)^3$, $p_2(q) = 3q(1-q)^2$, $p_3(q) = 3q^2(1-q)$, and $p_4(q) = q^3$.

The likelihood is $3^{n_2+n_3} \cdot (1-q)^{3n_1+2n_2+n_3} \cdot q^{n_2+2n_3+3n_4}$. Forming the log likelihood, taking the derivative with respect to q, equating to 0, and solving yields

$\hat{q} = \dfrac{n_2+2n_3+3n_4}{3n} = \dfrac{66+128+48}{480} = .504$. The estimated expected counts are $160(1-.504)^3 = 19.52$, $480(.504)(.496)^2 = 59.52$, 60.48, and 20.48, so

$$c^2 = \left[\frac{(14-19.52)^2}{19.52} + \ldots + \frac{(16-20.48)^2}{20.48}\right] = 1.56 + .71 + .20 + .98 = 3.45.$$

The number of degrees of freedom for the test is 4 – 1 – 1 = 2. H_o of a binomial distribution will be rejected using significance level .05 if $c^2 \geq c^2_{.05,2} = 5.992$. Because 3.45 < 5.992, H_o is not rejected, and the binomial model is judged to be quite plausible.

b. Now $\hat{q} = \dfrac{53}{150} = .353$ and the estimated expected counts are 13.54, 22.17, 12.09, and 2.20. The last estimated expected count is much less than 5, so the chi-squared test based on 2 d.f. should not be used.

13. According to the stated model, the three cell probabilities are $(1 - p)^2$, $2p(1 - p)$, and p^2, so we wish the value of p which maximizes $(1-p)^{2n_1}\left[2p(1-p)\right]^{n_2} p^{2n_3}$. Proceeding as in example 14.6 gives $\hat{p} = \dfrac{n_2 + 2n_3}{2n} = \dfrac{234}{2776} = .0843$. The estimated expected cell counts are then $n(1-\hat{p})^2 = 1163.85$, $n\left[2\hat{p}(1-\hat{p})\right]^2 = 214.29$, $n\hat{p}^2 = 9.86$. This gives

$$c^2 = \left[\frac{(1212 - 1163.85)^2}{1163.85} + \frac{(118 - 214.29)^2}{214.29} + \frac{(58 - 9.86)^2}{9.86}\right] = 280.3.$$

According to (14.15), H_o will be rejected if $c^2 \geq c^2_{a,2}$, and since $c^2_{.01,2} = 9.210$, H_o is soundly rejected; the stated model is strongly contradicted by the data.

14.

a. We wish to maximize $p^{\Sigma x_i - n}(1-p)^n$, or equivalently $(\Sigma x_i - n)\ln p + n\ln(1-p)$. Equating $\dfrac{d}{dp}$ to 0 yields $\dfrac{(\Sigma x_i - n)}{p} = \dfrac{n}{(1-p)}$, whence $p = \dfrac{(\Sigma x_i - n)}{\Sigma x_i}$. For the given data, $\Sigma x_i = (1)(1) + (2)(31) + \ldots + (12)(1) = 363$, so $\hat{p} = \dfrac{(363 - 130)}{363} = .642$, and $\hat{q} = .358$.

b. Each estimated expected cell count is $\hat{p}$ times the previous count, giving $n\hat{q} = 130(.358) = 46.54$, $n\hat{q}\hat{p} = 46.54(.642) = 29.88$, 19.18, 12.31, 17.91, 5.08, 3.26, … . Grouping all values ≥ 7 into a single category gives 7 cells with estimated expected counts 46.54, 29.88, 19.18, 12.31, 7.91, 5.08 (sum = 120.9), and 130 – 120.9 = 9.1. The corresponding observed counts are 48, 31, 20, 9, 6, 5, and 11, giving $c^2 = 1.87$. With k = 7 and m = 1 (p was estimated), from (14.15) we need $c^2_{.10,5} = 9.236$. Since 1.87 is not ≥ 9.236, we don't reject H_o.

15. The part of the likelihood involving q is $\left[(1-q)^4\right]^{n_1}\cdot\left[q(1-q)^3\right]^{n_2}\cdot\left[q^2(1-q)^2\right]^{n_3}\cdot$ $\left[q^3(1-q)\right]^{n_4}\cdot\left[q^4\right]^{n_5} = q^{n_2+2n_3+3n_4+4n_5}(1-q)^{4n_1+3n_2+2n_3+n_4} = q^{233}(1-q)^{367}$, so $\ln(likelihood) = 233\ln q + 367\ln(1-q)$. Differentiating and equating to 0 yields $\hat{q} = \frac{233}{600} = .3883$, and $(1-\hat{q}) = .6117$ [note that the exponent on q is simply the total # of successes (defectives here) in the n = 4(150) = 600 trials.] Substituting this q' into the formula for p_i yields estimated cell probabilities .1400, .3555, .3385, .1433, and .0227. Multiplication by 150 yields the estimated expected cell counts are 21.00, 53.33, 50.78, 21.50, and 3.41. the last estimated expected cell count is less than 5, so we combine the last two categories into a single one (≥ 3 defectives), yielding estimated counts 21.00, 53.33, 50.78, 24.91, observed counts 26, 51, 47, 26, and $c^2 = 1.62$. With d.f. = 4 – 1 – 1 = 2, since $1.62 < c^2_{.10,2} = 4.605$, the p-value > .10, and we do not reject H$_o$. The data suggests that the stated binomial distribution is plausible.

16. $\hat{l} = \bar{x} = \frac{(0)(6)+(1)(24)+(2)(42)+\ldots+(8)(6)+(9)(2)}{300} = \frac{1163}{300} = 3.88$, so the estimated cell probabilities are computed from $\hat{p} = e^{-3.88}\frac{(3.88)^x}{x!}$.

x	0	1	2	3	4	5	6	7	≥ 8
np(x)	6.2	24.0	46.6	60.3	58.5	45.4	29.4	16.3	13.3
obs	6	24	42	59	62	44	41	14	8

This gives $c^2 = 7.789$. To see whether the Poisson model provides a good fit, we need $c^2_{.10,9-1-1} = c^2_{.10,7} = 12.017$. Since $7.789 < 12.017$, the Poisson model does provide a good fit.

17. $\hat{l} = \frac{380}{120} = 3.167$, so $\hat{p} = e^{-3.167}\frac{(3.167)^x}{x!}$.

x	0	1	2	3	4	5	6	≥ 7
$\hat{p}$	.0421	.1334	.2113	.2230	.1766	.1119	.0590	.0427
$n\hat{p}$	5.05	16.00	25.36	26.76	21.19	13.43	7.08	5.12
obs	24	16	16	18	15	9	6	16

The resulting value of $c^2 = 103.98$, and when compared to $c^2_{.01,7} = 18.474$, it is obvious that the Poisson model fits very poorly.

18. $\hat{p}_1 = P(X < .100) = P\left(Z < \dfrac{.100 - .173}{.066}\right) = \Phi(-1.11) = .1335$,
$\hat{p}_2 = P(.100 \le X \le .150) = P(-1.11 \le Z \le -.35) = .2297$,
$\hat{p}_3 = P(-.35 \le Z \le .41) = .2959$, $\hat{p}_4 = P(.41 \le Z \le 1.17) = .2199$, and
$\hat{p}_5 = .1210$. The estimated expected counts are then (multiply $\hat{p}_i$ by n = 83) 11.08, 19.07, 24.56, 18.25, and 10.04, from which $c^2 = 1.67$. Comparing this with $c^2_{.05,5-1-2} = c^2_{.05,2} = 5.992$, the hypothesis of normality cannot be rejected.

19. With A = $2n_1 + n_4 + n_5$, B = $2n_2 + n_4 + n_6$, and C = $2n_3 + n_5 + n_6$, the likelihood is proportional to $q_1^A q_2^B (1 - q_1 - q_2)^C$, where A + B + C = 2n. Taking the natural log and equating both $\dfrac{\partial}{\partial q_1}$ and $\dfrac{\partial}{\partial q_2}$ to zero gives $\dfrac{A}{q_1} = \dfrac{C}{1 - q_1 - q_2}$ and $\dfrac{B}{q_2} = \dfrac{C}{1 - q_1 - q_2}$, whence $q_2 = \dfrac{Bq_1}{A}$. Substituting this into the first equation gives $q_1 = \dfrac{A}{A + B + C}$, and then $q_2 = \dfrac{B}{A + B + C}$. Thus $\hat{q}_1 = \dfrac{2n_1 + n_4 + n_5}{2n}$, $\hat{q}_2 = \dfrac{2n_2 + n_4 + n_6}{2n}$, and $\left(1 - \hat{q}_1 - \hat{q}_2\right) = \dfrac{2n_3 + n_5 + n_6}{2n}$. Substituting the observed n_I's yields $\hat{q}_1 = \dfrac{2(49) + 20 + 53}{400} = .4275$, $\hat{q}_2 = \dfrac{110}{400} = .2750$, and $\left(1 - \hat{q}_1 - \hat{q}_2\right) = .2975$, from which $\hat{p}_1 = (.4275)^2 = .183$, $\hat{p}_2 = .076$, $\hat{p}_3 = .089$, $\hat{p}_4 = 2(.4275)(.275) = .235$, $\hat{p}_5 = .254$, $\hat{p}_6 = .164$.

Category	**1**	**2**	**3**	**4**	**5**	**6**
np	36.6	15.2	17.8	47.0	50.8	32.8
observed	49	26	14	20	53	38

This gives $c^2 = 29.1$. With $c^2_{.01,6-1-2} = c^2_{.01,3} = 11.344$, and $c^2_{.01,6-1} = c^2_{.01,5} = 15.085$, according to (14.15) H_o must be rejected since $29.1 \ge 15.085$.

20. The pattern of points in the plot appear to deviate from a straight line, a conclusion that is also supported by the small p-value (< .01000) of the Ryan-Joiner test. Therefore, it is implausible that this data came from a normal population. In particular, the observation 116.7 is a clear outlier. It would be dangerous to use the one-sample t interval as a basis for inference.

21. The Ryan-Joiner test p-value is larger than .10, so we conclude that the null hypothesis of normality cannot be rejected. This data could reasonably have come from a normal population. This means that it would be legitimate to use a one-sample t test to test hypotheses about the true average ratio.

22.

x_i	y_i	x_i	y_i	x_i	y_i
69.5	-1.967	75.5	-.301	79.6	.634
71.9	-1.520	75.7	-.199	79.7	.761
72.6	-1.259	75.8	-.099	79.9	.901
73.1	-1.063	76.1	.000	80.1	1.063
73.3	-.901	76.2	.099	82.2	1.259
73.5	-.761	76.9	.199	83.7	1.520
74.1	-.634	77.0	.301	93.7	1.967
74.2	-.517	77.9	.407		
75.3	-.407	78.1	.517		

n.b.: Minitab was used to calculate the y_I's. $\Sigma x_{(i)} = 1925.6$, $\Sigma x_{(i)}^2 = 148{,}871$, $\Sigma y_i = 0$, $\Sigma y_i^2 = 22.523$, $\Sigma x_{(i)} y_i = 103.03$, so

$$r = \frac{25(103.03)}{\sqrt{25(148{,}871) - (1925.6)^2}\sqrt{25(25.523)}} = .923$$

. Since $c_{.01} = .9408$, and $.923 < .9408$, even at the very smallest significance level of .01, the null hypothesis of population normality must be rejected (the largest observation appears to be the primary culprit).

23. Minitab gives r = .967, though the hand calculated value may be slightly different because when there are ties among the $x_{(i)}$'s, Minitab uses the same y_I for each $x_{(i)}$ in a group of tied values. $C_{10} = .9707$, and $c_{.05} = 9639$, so $.05 < \text{p-value} < .10$. At the 5% significance level, one would have to consider population normality plausible.

Section 14.3

24. H_o: TV watching and physical fitness are independent of each other
H_a: the two variables are not independent
Df = (4 – 1)(2 – 1) = 3
With $a = .05$, RR: $c^2 \geq 7.815$
Computed $c^2 = 6.161$
Fail to reject H_o. The data fail to indicate an association between daily TV viewing habits and physical fitness.

25. Let P_{ij} = the proportion of white clover in area of type i which has a type j mark (i = 1, 2; j = 1, 2, 3, 4, 5). The hypothesis H_o: $p_{1j} = p_{2j}$ for j = 1, …, 5 will be rejected at level .01 if $c^2 \ge c^2_{.01,(2-1)(5-1)} = c^2_{.01,4} = 13.277$.

$\hat{E}_{ij}$	1	2	3	4	5		
1	449.66	7.32	17.58	8.79	242.65	726	$c^2 = 23.18$
2	471.34	7.68	18.42	9.21	254.35	761	
	921	15	36	18	497	1487	

Since $23.18 \ge 13.277$, H_o is rejected.

26. Let p_{i1} = the probability that a fruit given treatment i matures and p_{i2} = the probability that a fruit given treatment i aborts. Then H_o: $p_{i1} = p_{i2}$ for i = 1, 2, 3, 4, 5 will be rejected if $c^2 \ge c^2_{.01,4} = 13.277$.

Observed		Estimated Expected		
Matured	Aborted	Matured	Aborted	n_i
141	206	110.7	236.3	347
28	69	30.9	66.1	97
25	73	31.3	66.7	98
24	78	32.5	69.5	102
20	82	32.5	69.5	102
		238	508	746

Thus $c^2 = \frac{(141-110.7)^2}{110.7} + \ldots + \frac{(82-69.5)^2}{69.5} = 24.82$, which is ≥ 13.277, so H_o is rejected at level .01.

27. With i = 1 identified with men and i = 2 identified with women, and j = 1, 2, 3 denoting the 3 categories L>R, L=R, L<R, we wish to test H_o: $p_{1j} = p_{2j}$ for j = 1, 2, 3 vs. H_a: p_{1j} not equal to p_{2j} for at least one j. The estimated cell counts for men are 17.95, 8.82, and 13.23 and for women are 39.05, 19.18, 28.77, resulting in $c^2 = 44.98$. With (2 – 1)(3 – 1) = 2 degrees of freedom, since $44.98 > c^2_{.005,2} = 10.597$, p-value < .005, which strongly suggests that H_o should be rejected.

28. With p_{ij} denoting the probability of a type j response when treatment i is applied, H_o: $p_{1j} = p_{2j} = p_{3j} = p_{4j}$ for j = 1, 2, 3, 4 will be rejected at level .005 if $c^2 \geq c^2_{.005,9} = 23.587$.

$\hat{E}_{ij}$	1	2	3	4
1	24.1	10.0	21.6	40.4
2	25.8	10.7	23.1	43.3
3	26.1	10.8	23.4	43.8
4	30.1	12.5	27.0	50.5

$c^2 = 27.66 \geq 23.587$, so reject H_o at level .005

29. H_o: $p_{1j} = \ldots = p_{6j}$ for j = 1, 2, 3 is the hypothesis of interest, where p_{ij} is the proportion of the j^{th} sex combination resulting from the i^{th} genotype. H_o will be rejected at level .10 if $c^2 \geq c^2_{.10,10} = 15.987$.

$\hat{E}_{ij}$	1	2	3	
1	35.8	83.1	35.1	154
2	39.5	91.8	38.7	170
3	35.1	81.5	34.4	151
4	9.8	22.7	9.6	42
5	5.1	11.9	5.0	22
6	26.7	62.1	26.2	115
	152	353	149	654

c^2	1	2	3	
	.02	.12	.44	
	.06	.66	1.01	
	.13	.37	.34	
	.32	.49	.26	
	.00	.06	.19	
	.40	.14	1.47	
				6.46

(carrying 2 decimal places in $\hat{E}_{ij}$ yields $c^2 = 6.49$). Since 6.46 < 15.987, H_o cannot be rejected at level .10.

30. H_o: the design configurations are homogeneous with respect to type of failure vs. H_a: the design configurations are not homogeneous with respect to type of failure.

$\hat{E}_{ij}$	1	2	3	4	
1	16.11	43.58	18.00	12.32	90
2	7.16	19.37	8.00	5.47	40
3	10.74	29.05	12.00	8.21	60
	34	92	38	26	190

$\chi^2 = \dfrac{(20-16.11)^2}{16.11} + \ldots + \dfrac{(5-8.21)^2}{8.21} = 13.253$. With 6 df,

$\chi^2_{.05,6} = 12.592 < 13.253 < \chi^2_{.025,6} = 14.440$, so .025 < p-value < .05. Since the p-value is < .05, we reject H_o. (If a smaller significance level were chosen, a different conclusion would be reached.) Configuration appears to have an effect on type of failure.

31. With I denoting the I^{th} type of car (I = 1, 2, 3, 4) and j the j^{th} category of commuting distance, H_o: $p_{ij} = p_{i.}\, p_{.j}$ (type of car and commuting distance are independent) will be rejected at level .05 if $\chi^2 \geq \chi^2_{.05,6} = 12.592$.

$\hat{E}_{ij}$	1	2	3	
1	10.19	26.21	15.60	52
2	11.96	30.74	18.30	61
3	19.40	49.90	29.70	99
4	7.45	19.15	11.40	38
	49	126	75	250

$\chi^2 = 14.15 \geq 12.592$, so the independence hypothesis H_o is rejected at level .05 (but not at level .025!)

32. $$\chi^2 = \frac{(479-494.4)^2}{494.4} + \frac{(173-151.5)^2}{151.5} + \frac{(119-125.2)^2}{125.2} + \frac{(214-177.0)^2}{177.0} + \frac{(47-54.2)^2}{54.2}$$
$$= \frac{(15-44.8)^2}{44.8} + \frac{(172-193.6)^2}{193.6} + \frac{(45-59.3)^2}{59.3} + \frac{(85-49.0)^2}{49.0} = 64.65 \geq \chi^2_{.01,4} = 13.277$$
so the independence hypothesis is rejected in favor of the conclusion that political views and level of marijuana usage are related.

33. $c^2 = \Sigma\Sigma \frac{\left(N_{ij} - \hat{E}_{ij}\right)^2}{\hat{E}_{ij}} = \Sigma\Sigma \frac{N_{ij}^2 - 2\hat{E}_{ij}N_{ij} + \hat{E}_{ij}^2}{\hat{E}_{ij}} = \frac{\Sigma\Sigma N_{ij}^2}{\hat{E}_{ij}} - 2\Sigma\Sigma N_{ij} + \Sigma\Sigma\hat{E}_{ij}$, but

$\Sigma\Sigma\hat{E}_{ij} = \Sigma\Sigma N_{ij} = n$, so $c^2 = \Sigma\Sigma \frac{N_{ij}^2}{\hat{E}_{ij}} - n$. This formula is computationally efficient because there is only one subtraction to be performed, which can be done as the last step in the calculation.

34. This is a $3 \times 3 \times 3$ situation, so there are 27 cells. Only the total sample size n is fixed in advance of the experiment, so there are 26 freely determined cell counts. We must estimate $p_{..1}, p_{..2}, p_{..3}, p_{.1.}, p_{.2.}, p_{.3.}, p_{1..}, p_{2..}$, and $p_{3..}$, but $\Sigma p_{i..} = \Sigma p_{.j.} = \Sigma p_{..k} = 1$ so only 6 independent parameters are estimated. The rule for d.f. now gives c^2 df = 26 – 6 = 20.

35. With p_{ij} denoting the common value of p_{ij1}, p_{ij2}, p_{ij3}, p_{ij4} (under H_o), $\hat{p}_{ij} = \frac{N_{ij}}{n}$ and

$\hat{E}_{ijk} = \frac{n_k N_{ij}}{n}$. With four different tables (one for each region), there are 8 + 8 + 8 + 8 = 32 freely determined cell counts. Under H_o, $p_{11}, \ldots, p_{33}$ must be estimated but $\Sigma\Sigma p_{ij} = 1$ so only 8 independent parameters are estimated, giving c^2 df = 32 – 8 = 24.

36.

a.

Observed

13	19	28	60
7	11	22	40
20	30	50	100

Estimated Expected

12	18	30
8	12	20

$c^2 = \frac{(13-12)^2}{12} + \ldots + \frac{(22-20)^2}{20} = .6806$. Because $.6806 < c^2_{.10,2} = 4.605$, H_o is not rejected.

b. Each observation count here is 10 times what it was in **a**, and the same is true of the estimated expected counts so now $c^2 = 6.806 \geq 4.605$, and H_o is rejected. With the much larger sample size, the departure from what is expected under H_o, the independence hypothesis, is statistically significant – it cannot be explained just by random variation.

c. The observed counts are .13n, .19n, .28n, .07n, .11n, .22n, whereas the estimated expected $\frac{(.60n)(.20n)}{n} =$ .12n, .18n, .30n, .08n, .12n, .20n, yielding $c^2 = .006806n$. H_o will be rejected at level .10 iff $.006806n \geq 4.605$, i.e., iff $n \geq 676.6$, so the minimum n = 677.

Supplementary Exercises

37. There are 3 categories here – firstborn, middleborn, (2^{nd} or 3^{rd} born), and lastborn. With p_1, p_2, and p_3 denoting the category probabilities, we wish to test H_o: $p_1 = .25$, $p_2 = .50$ (p_2 = P(2^{nd} or 3^{rd} born) = .25 + .25 = .50), $p_3 = .25$. H_o will be rejected at significance level .05 if $c^2 \ge c^2_{.05,2} = 5.992$. The expected counts are (31)(.25) = 7.75, (31)(.50) = 15.5, and 7.75, so $c^2 = \frac{(12-7.75)^2}{7.75} + \frac{(11-15.5)^2}{15.5} + \frac{(8-7.75)^2}{7.75} = 3.65$. Because 3.65 < 5.992, H_o is not rejected. The hypothesis of equiprobable birth order appears quite plausible.

38. Let p_{i1} = the proportion of fish receiving treatment i (i = 1, 2, 3) who are parasitized. We wish to test H_o: $p_{1j} = p_{2j} = p_{3j}$ for j = 1, 2. With df = (2 – 1)(3 – 1) = 2, H_o will be rejected at level .01 if $c^2 \ge c^2_{.01,2} = 9.210$.

Observed			Estimated Expected	
30	3	33	22.99	10.01
16	8	24	16.72	7.28
16	16	32	22.29	9.71
62	27	89		

This gives $c^2 = 13.1$. Because $13.1 \ge 9.210$, H_o should be rejected. The proportion of fish that are parasitized does appear to depend on which treatment is used.

39. H_o: gender and years of experience are independent; H_a: gender and years of experience are not independent. Df = 4, and we reject H_o if $c^2 \ge c^2_{.01,4} = 13.277$.

	Years of Experience				
Gender	1 – 3	4 – 6	7 – 9	10 – 12	13 +
Male Observed	202	369	482	361	811
Expected	285.56	409.83	475.94	347.04	706.63
$\frac{(O-E)^2}{E}$	24.451	4.068	.077	.562	15.415
Female Observed	230	251	238	164	258
Expected	146.44	210.17	244.06	177.96	362.37
$\frac{(O-E)^2}{E}$	47.680	7.932	.151	1.095	30.061

$c^2 = \Sigma\frac{(O-E)^2}{E} = 131.492$. Reject H_o. The two variables do not appear to be independent. In particular, women have higher than expected counts in the beginning category (1 – 3 years) and lower than expected counts in the more experienced category (13+ years).

40.

a. H_o: The probability of a late-game leader winning is independent of the sport played; H_a: The two variables are not independent. With 3 df, the computed $\chi^2 = 10.518$, and the p-value < .015 is also < .05, so we would reject H_o. There appears to be a relationship between the late-game leader winning and the sport played.

b. Quite possibly: Baseball had many fewer than expected late-game leader losses.

41. The null hypothesis H_o: $p_{ij} = p_{i.}\, p_{.j}$ states that level of parental use and level of student use are independent in the population of interest. The test is based on (3 – 1)(3 – 1) = 4 df.

Estimated Expected

119.3	57.6	58.1	235
82.8	33.9	40.3	163
23.9	11.5	11.6	47
226	109	110	445

The calculated value of $\chi^2 = 22.4$. Since $22.4 > \chi^2_{.005,4} = 14.860$, p-value < .005, so H_o should be rejected at any significance level greater than .005. Parental and student use level do not appear to be independent.

42. The estimated expected counts are displayed below, from which $\chi^2 = 197.70$. A glance at the 6 df row of Table A.7 shows that this test statistic value is highly significant – the hypothesis of independence is clearly implausible.

Estimated Expected

	Home	Acute	Chronic	
15 – 54	90.2	372.5	72.3	535
55 – 64	113.6	469.3	91.1	674
65 – 74	142.7	589.0	114.3	846
> 74	157.5	650.3	126.2	934
	504	2081	404	2989

43. This is a test of homogeneity: H_o: $p_{1j} = p_{2j} = p_{3j}$ for j = 1, 2, 3, 4, 5. The given SPSS output reports the calculated $c^2 = 70.64156$ and accompanying p-value (significance) of .0000. We reject H_o at any significance level. The data strongly supports that there are differences in perception of odors among the three areas.

44. The accompanying table contains both observed and estimated expected counts, the latter in parentheses.

	Age					
Want	127 (131.1)	118 (123.3)	77 (71.7)	61 (55.1)	41 (42.8)	424
Don't	23 (18.9)	23 (17.7)	5 (10.3)	2 (7.9)	8 (6.2)	61
	150	141	82	63	49	485

This gives $c^2 = 11.60 \geq c^2_{.05,4} = 9.488$. At level .05, the null hypothesis of independence is rejected, though it would not be rejected at the level .01 (.01 < p-value < .025).

45. $(n_1 - np_{10})^2 = (np_{10} - n_1)^2 = (n - n_1 - n(1 - p_{10}))^2 = (n_2 - np_{20})^2$. Therefore

$$c^2 = \frac{(n_1 - np_{10})^2}{np_{10}} + \frac{(n_2 - np_{20})^2}{np_{20}} = \frac{(n_1 - np_{10})^2}{n_2}\left(\frac{n}{p_{10}} + \frac{n}{p_{20}}\right)$$

$$= \left(\frac{n_1}{n} - p_{10}\right)^2 \cdot \left(\frac{n}{p_{10}p_{20}}\right) = \frac{(\hat{p}_1 - p_{10})^2}{p_{10}p_{20}/n} = z^2.$$

46.

a.

obsv	22	10	5	11
exp	13.189	10	7.406	17.405

H_0: probabilities are as specified.
H_a: probabilities are not as specified.

Test Statistic: $c^2 = \frac{(22 - 13.189)^2}{13.189} + \frac{(10-10)^2}{10} + \frac{(5 - 7.406)^2}{7.406} + \frac{(11 - 17.405)^2}{17.405}$

$= 5.886 + 0 + 0.782 + 2.357 = 9.025$. Rejection Region: $c^2 > c^2_{.05,2} = 5.99$

Since 9.025 > 5.99, we reject H_0. The model postulated in the exercise is not a good fit.

b.

p_i	0.45883	0.18813	0.11032	0.24272
exp	22.024	9.03	5.295	11.651

$$c^2 = \frac{(22-22.024)^2}{22.024} + \frac{(10-9.03)^2}{9.03} + \frac{(5-5.295)^2}{5.295} + \frac{(11-11.651)^2}{11.651}$$
$$= .0000262 + .1041971 + .0164353 + .0363746 = .1570332$$

With the same rejection region as in a, we do not reject the null hypothesis. This model does provide a good fit.

47.

a. Our hypotheses are H_0: no difference in proportion of concussions among the three groups. Vs H_a: there is a difference …

Observed	Concussion	No Concussion	Total
Soccer	45	46	91
Non Soccer	28	68	96
Control	8	45	53
Total	81	159	240

Expected	Concussion	No Concussion	Total
Soccer	30.7125	60.2875	91
Non Soccer	32.4	63.6	96
Control	17.8875	37.1125	53
Total	81	159	240

$$c^2 = \frac{(45-30.7125)^2}{30.7125} + \frac{(46-60.2875)^2}{60.2875} + \frac{(28-32.4)^2}{32.4} + \frac{(68-63.6)^2}{63.6}$$
$$+ \frac{(8-17.8875)^2}{17.8875} + \frac{(45-37.1125)^2}{37.1125} = 19.1842$$

The df for this test is (I – 1)(J – 1) = 2, so we reject H_o if $c^2 > c^2_{.05,2} = 5.99$. 19.1842 > 5.99, so we reject H_0. There is a difference in the proportion of concussions based on whether a person plays soccer.

b. We are testing the hypothesis H_0: ρ = 0 vs H_a: ρ ? 0. The test statistic is

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{-.22\sqrt{89}}{\sqrt{1-.22^2}} = -2.13$$

At significance level α = .01, we would fail to reject and conclude that there is no evidence of non-zero correlation in the population. If we were willing to accept a higher significance level, our decision could change. At best, there is evidence of only weak correlation.

c. We will test to see if the average score on a controlled word association test is the same for soccer and non-soccer athletes. H_0: $\mu_1 = \mu_2$ vs H_a: μ_1 ? μ_2 . We'll use test statistic $t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}}$. With $\frac{s_1^2}{m} = 3.206$ and $\frac{s_2^2}{n} = 1.854$,

$t = \frac{(37.50 - 39.63)}{\sqrt{3.206 + 1.854}} = -.95$. The df = $\frac{(3.206 + 1.854)^2}{\frac{3.206^2}{25} + \frac{1.854^2}{55}} \approx 56$. The p-value will be > .10, so we do not reject H_0 and conclude that there is no difference in the average score on the test for the two groups of athletes.

d. Our hypotheses for ANOVA are H_0: all means are equal vs H_a: not all means are equal. The test statistic is $f = \frac{MSTr}{MSE}$.

$$SSTr = 91(.30 - .35)^2 + 96(.49 - .35)^2 + 53(.19 - .35)^2 = 3.4659$$

$$MSTr = \frac{3.4659}{2} = 1.73295$$

$SSE = 90(.67)^2 + 95(.87)^2 + 52(.48)^2 = 124.2873$ and

$MSE = \frac{124.2873}{237} = .5244$. Now, $f = \frac{1.73295}{.5244} = 3.30$. Using df 2,200 from table A.9, the p value is between .01 and .05. At significance level .05, we reject the null hypothesis. There is sufficient evidence to conclude that there is a difference in the average number of prior non-soccer concussions between the three groups.

48.

a. H_o: $p_0 = p_1 = \ldots = p_9 = .10$ vs H_a: at least one p_i ? .10, with df = 9.

b. H_o: $p_{ij} = .01$ for I and j= 1,2,…,9 vs H_a: at least one p_{ij} ? 0, with df = 99.

c. For this test, the number of p's in the Hypothesis would be 10^5 = 100,000 (the number of possible combinations of 5 digits). Using only the first 100,000 digits in the expansion, the number of non-overlapping groups of 5 is only 20,000. We need a much larger sample size!

d. Based on these p-values, we could conclude that the digits of p behave as though they were randomly generated.